A Primer on Causal Diagram Learning

— Interpreting Causation from the Causal Discovery Perspective

Xuanzhi Chen *March 10, 2024*

xuanzhichen.42@gmail.com / xuanzhichen.github.io

"As X-rays are to the surgeon, causal diagrams are for causation." — Judea Pearl

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因果图学习入门

A Primer on Causal Diagram Learning

—基于因果发现的视角阐述因果关系

陈炫志　March 10. 2024

xuanzhichen.42@gmail.com / xuanzhichen.github.io

"因果图之于因果关系，就犹如X-射线之于外科医生" — Judea Pearl （因果模型之父）

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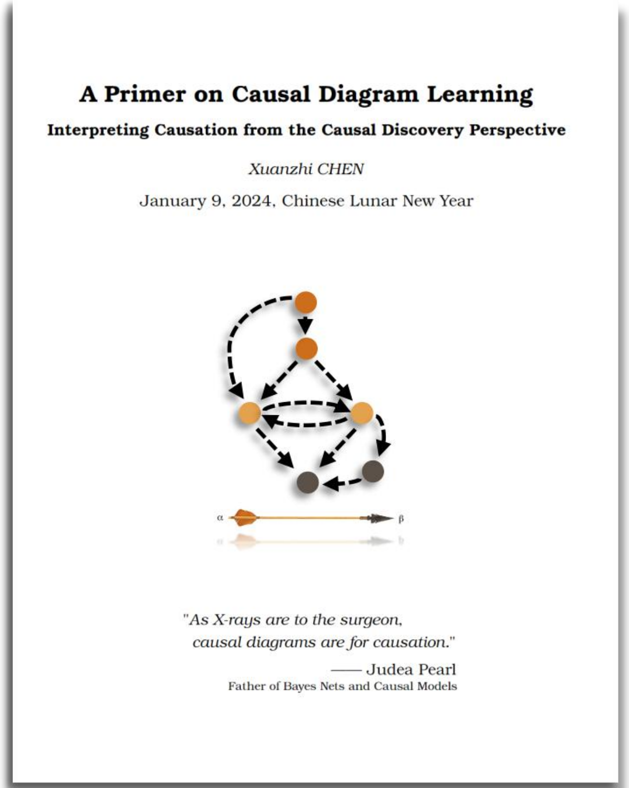
Acknowledgements

- **Acknowledgements:** *DMIR Lab, China*
  - **D**ata **M**ining and **I**nformation **R**etrieval Laboratory
  - Causal Discovery and Causality-Related Learning
  - Director: Prof. Cai (Ruichu Cai)

- **My Undergraduate Mentor:** *Wei Chen*
  - *"I would not have completed this article during my busy graduation season but for remembering the encouragement from her."*



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Xuanzhi CHEN

January 9, 2024, Chinese Lunar New Year

"As X-rays are to the surgeon,
causal diagrams are for causation."

—— Judea Pearl

Father of Bayes Nets and Causal Models

Acknowledgements

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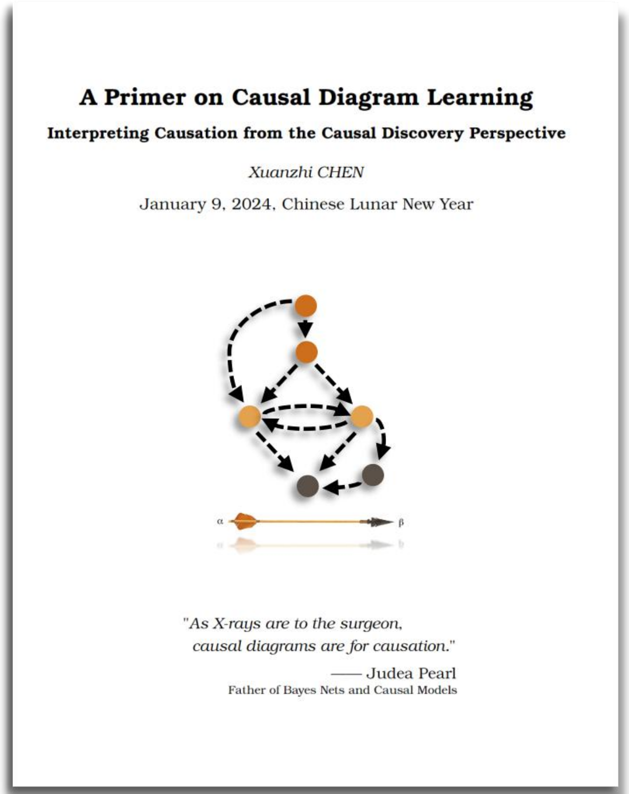
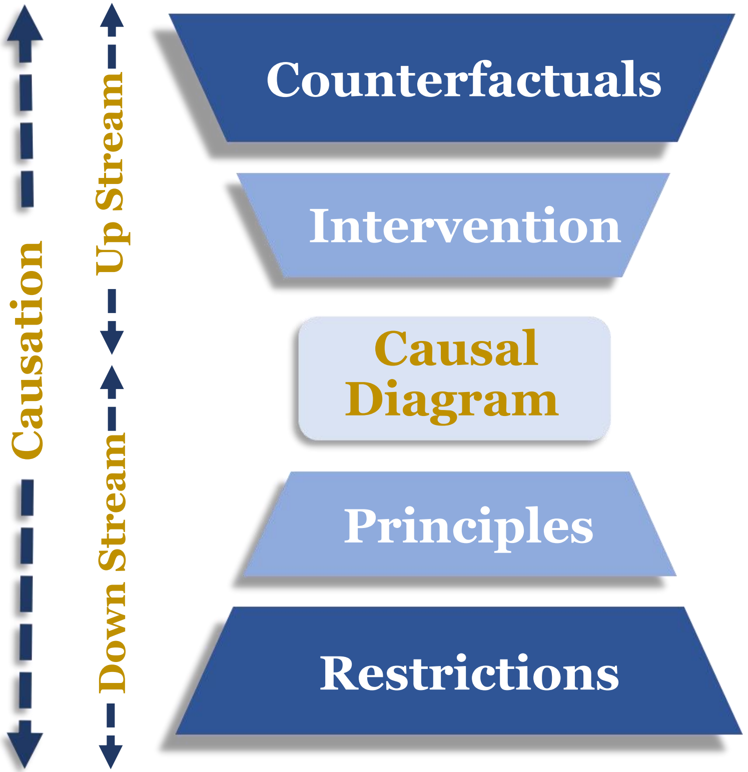
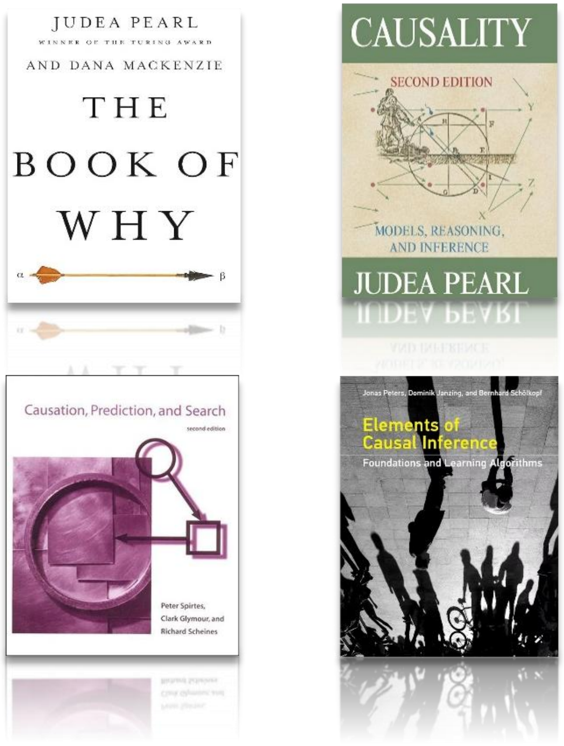
  - Causal Discovery and Causality-Related Learning
  - Director: Prof. Cai （蔡瑞初）
- **My Undergraduate Mentor: 陈薇**
  - “在繁忙的毕业季里，我本很可能无法坚持完成关于这段因果发现入门手册的写作——但我一直记着两年前当我最开始跟着她研究因果关系时她对我的鼓励”。



Clue to Celebrated Books as for Causal Diagram Learning





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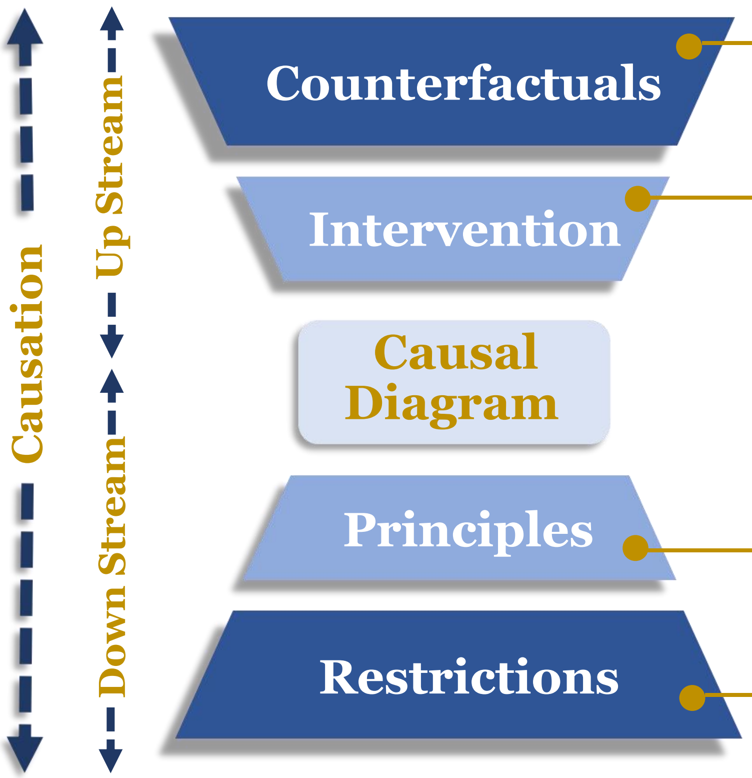
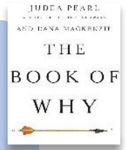
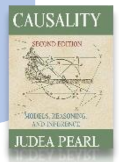


Up Stream: Functionality

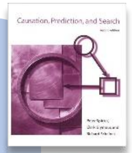
- ✓ The Ladder of Causation
- ✓ Preemption and Causal Beam
- ✓ Do Operation and Causal Effect
- ✓ …

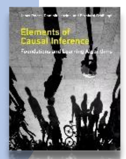




Down Stream: Learning

- ✓ Deterministic System
- ✓ Minimality and Faithfulness
- ✓ Function Capacity
- ✓ Inducing Path
- ✓ …

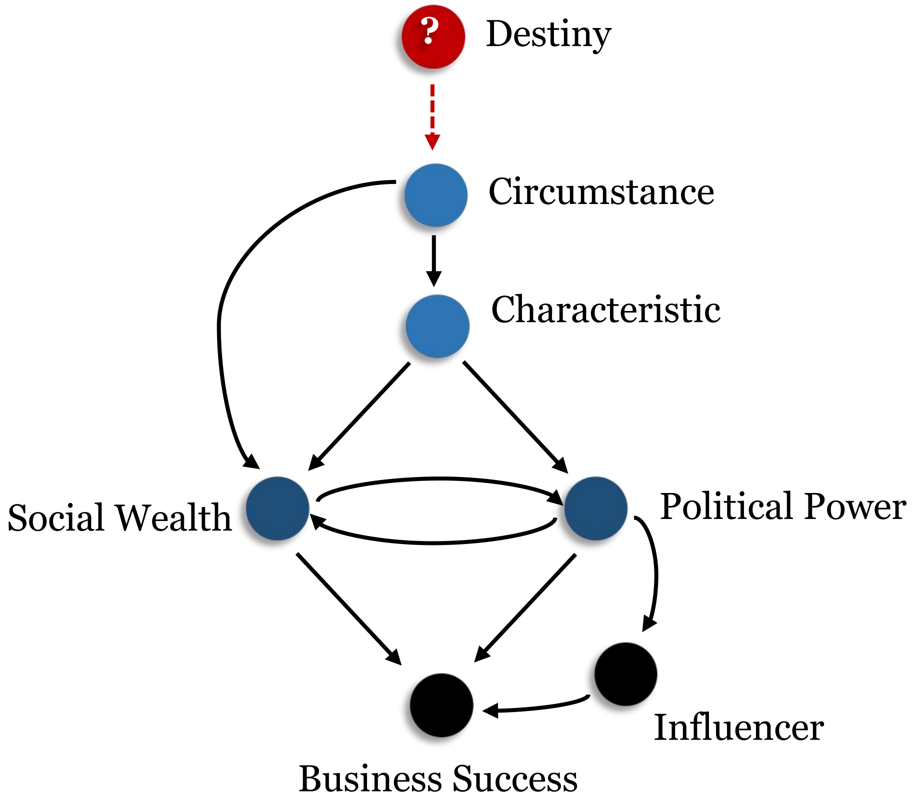




CONTENT

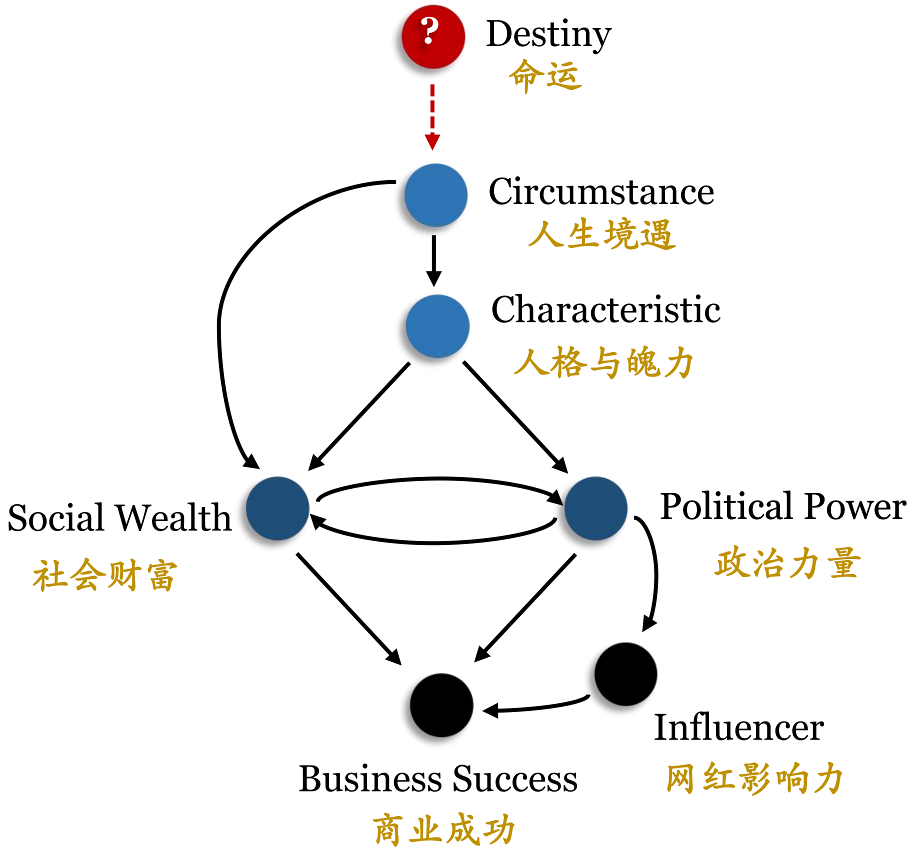
- Acknowledgements
- Introduction
  - Why Did I Write the Primer
  - An Overview of Causal Diagrams
  - Structure of This Series
- Source

Over the Causal Diagram: Causal Model



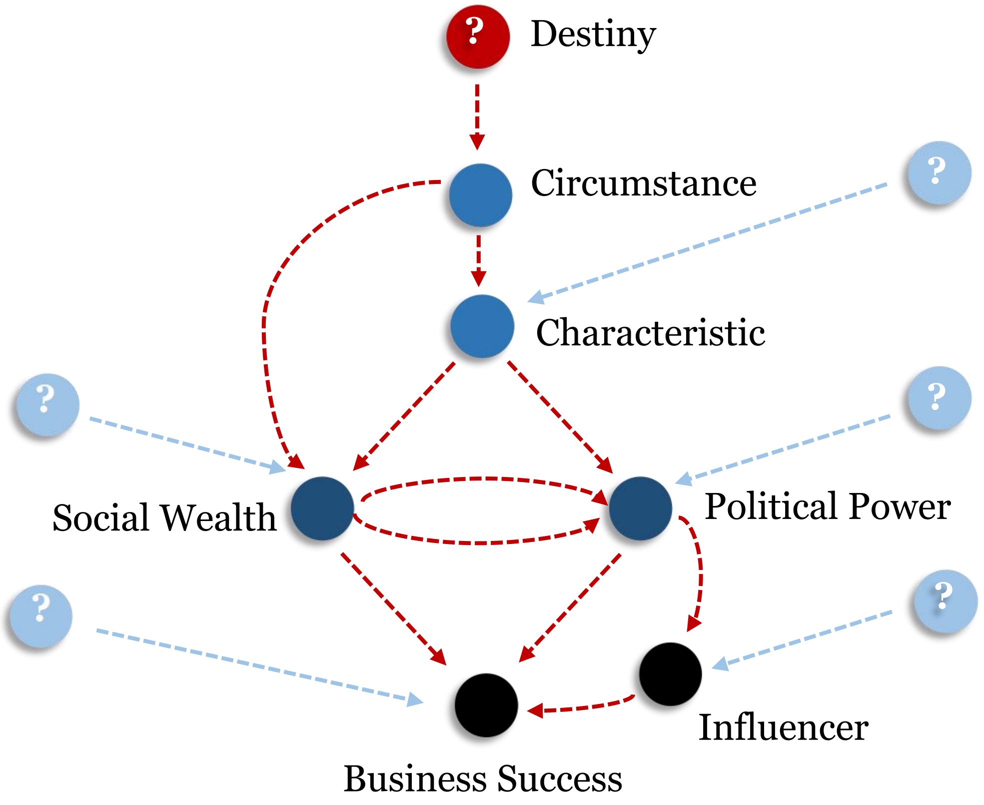
"Does God Play Dice and Conquer the Causation over Our Destiny ?"

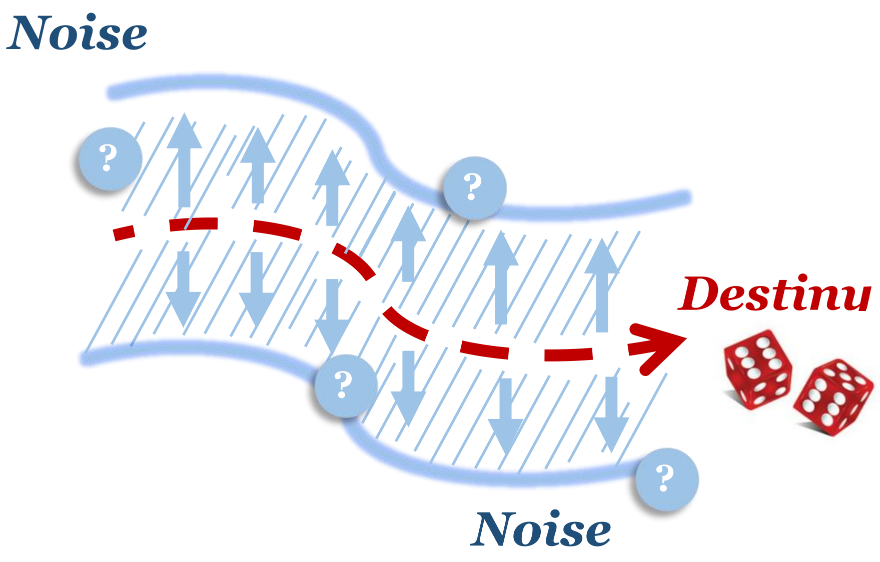
Over the Causal Diagram: Causal Model



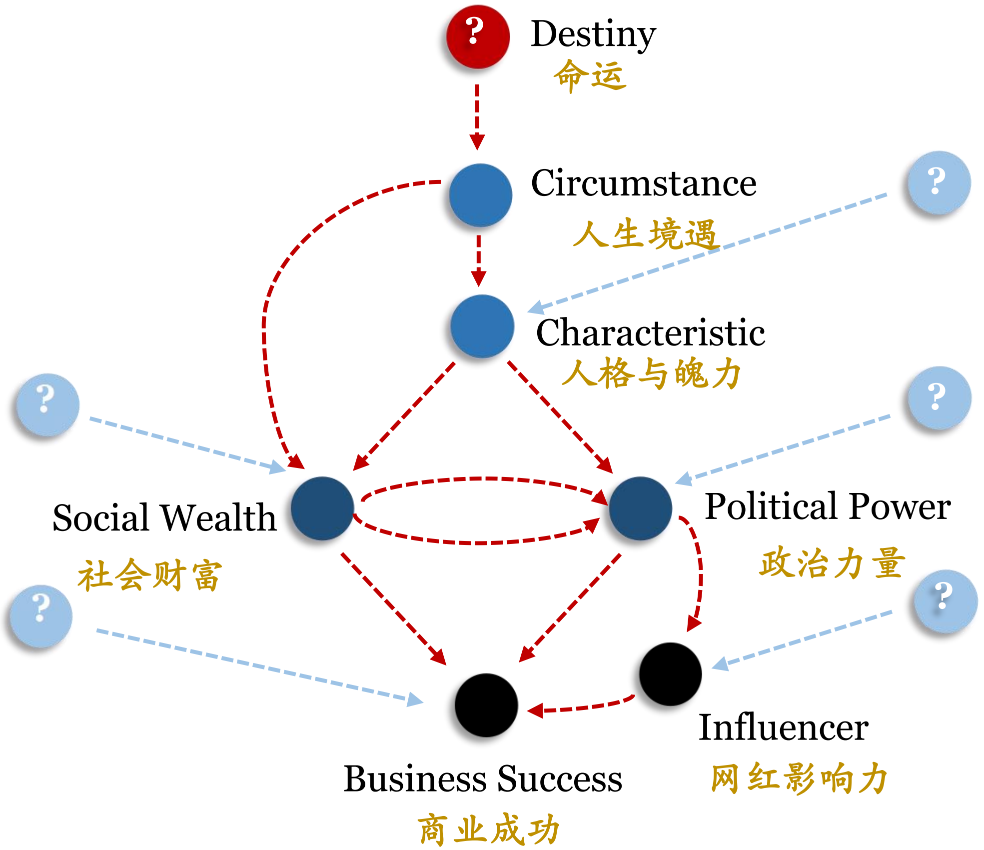
"是上帝掷了一颗骰子并掌握着我们一生中的因果关系吗？"

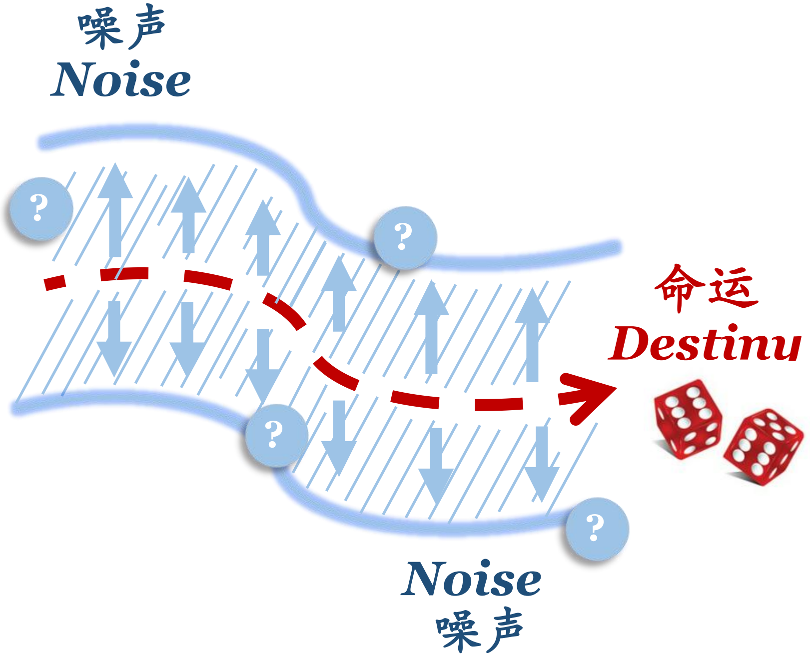
Over the Causal Diagram: Causal Model



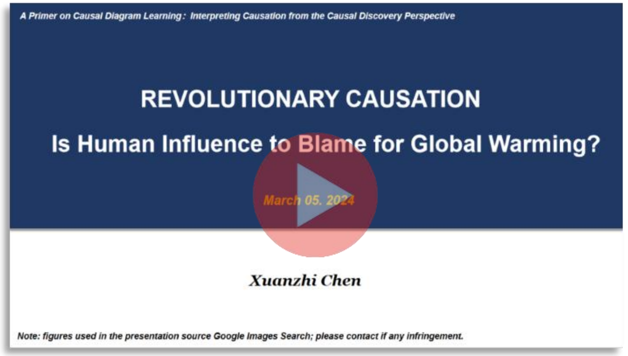


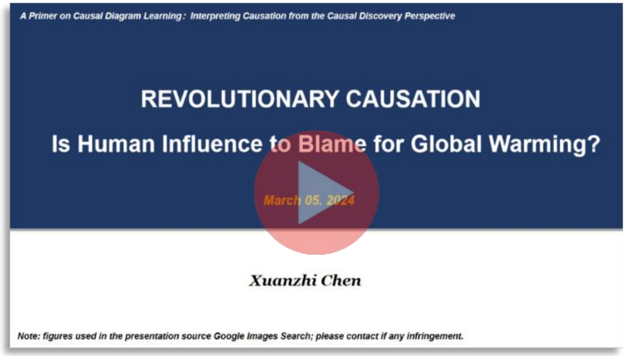
Over the Causal Diagram: Causal Model

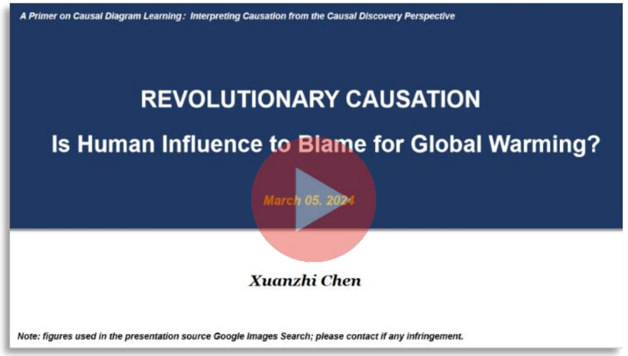




Short Four-Episodes







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REVOLUTIONARY CAUSATION

Is Human Influence to Blame for Global Warming?

March 05. 2024

Xuanzhi Chen

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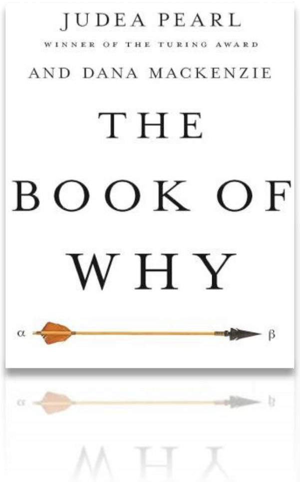
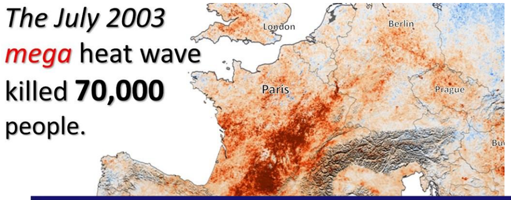
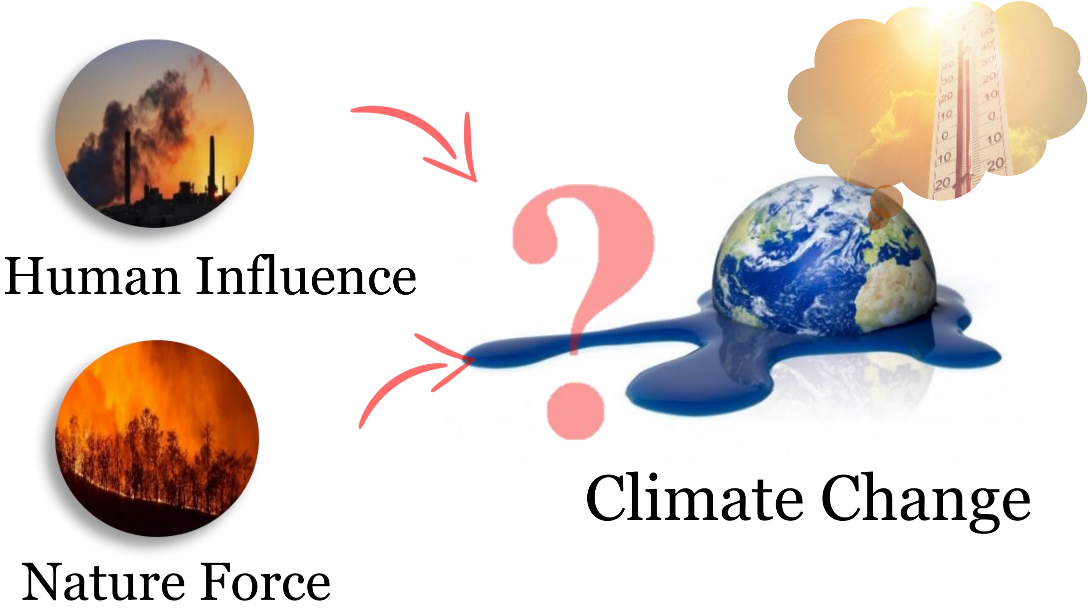
Why Does Focusing on a Statement is Important?







*"It is **very likely** that **over half** the FAR (Fraction of Attribution Risk) of European summer temperature anomalies is attributable to human influence."*

—Dr. Allen

A Language for Analysis of Causes?

Envision the (counterfactual) worlds where…

Probability of Necessity (PN):

Probability of Sufficiency (PS):

*"It is **very likely** that **over half** the FAR (Fraction of Attribution Risk) of European summer temperature anomalies is attributable to human influence."*

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*"It is **very likely** that **over half** the FAR (Fraction of Attribution Risk) of European summer temperature anomalies is **attributable to human influence**."*

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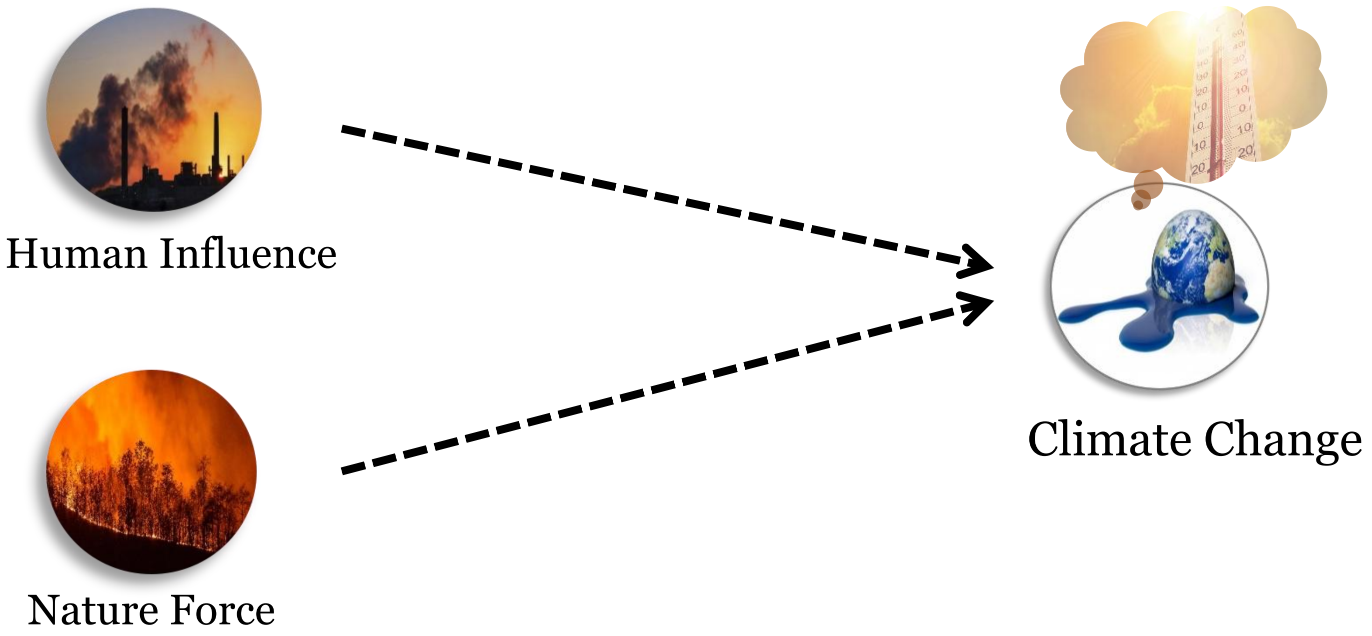
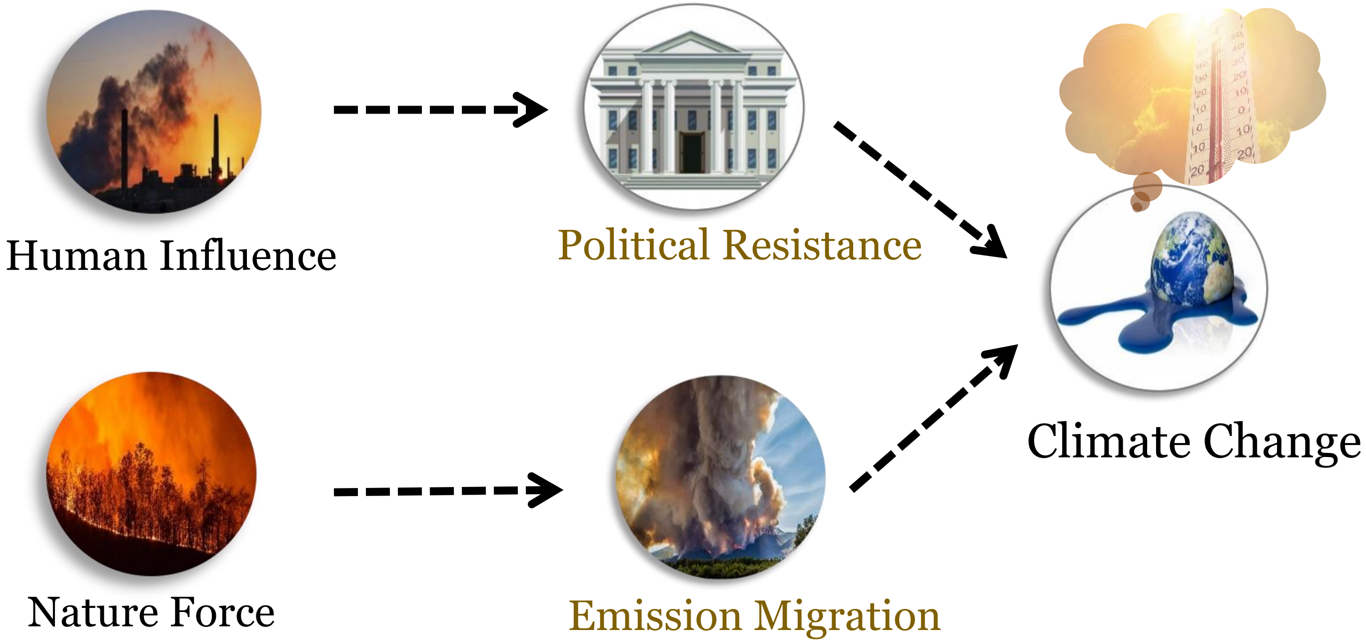
Insufficiency in Analysis of Causes ?

A Causal Diagram Describing Causal Insufficiency



A Causal Diagram Describing Causal Insufficiency



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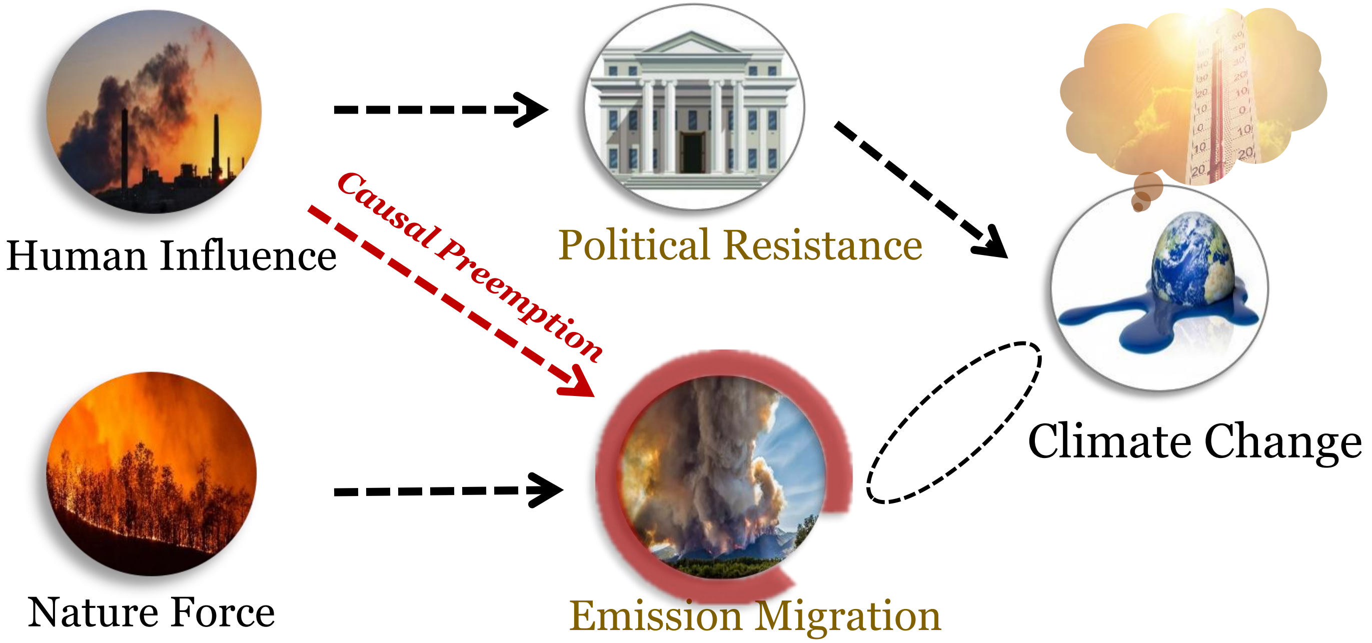
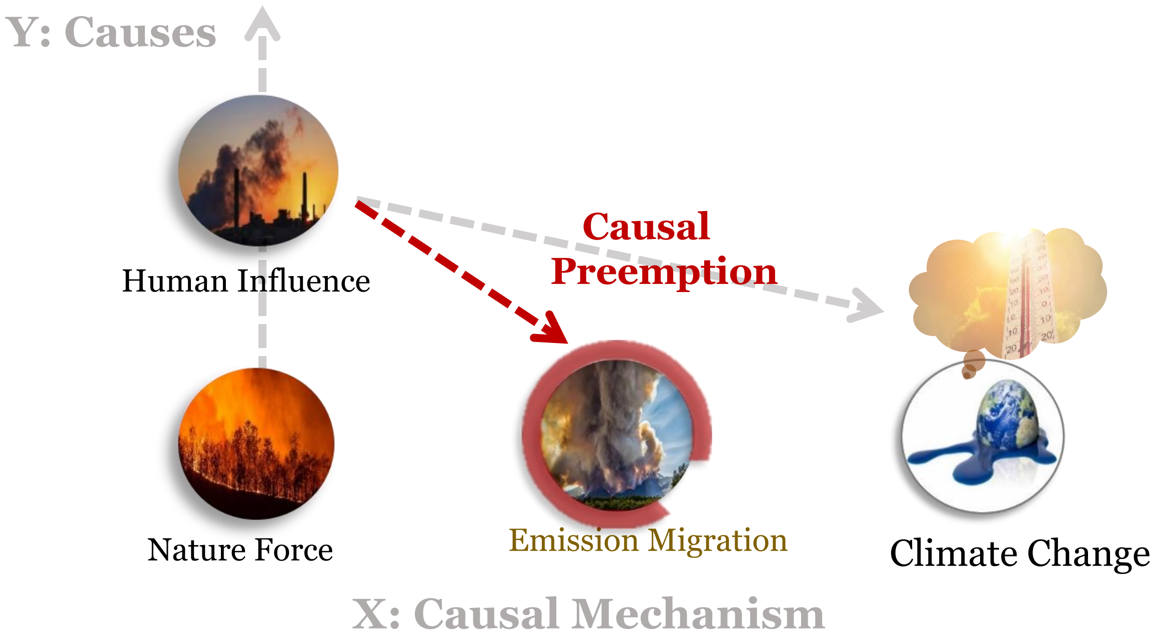


*"Techniques such as **drones** and satellites manufactured by the industries can also **protect** our climate."*

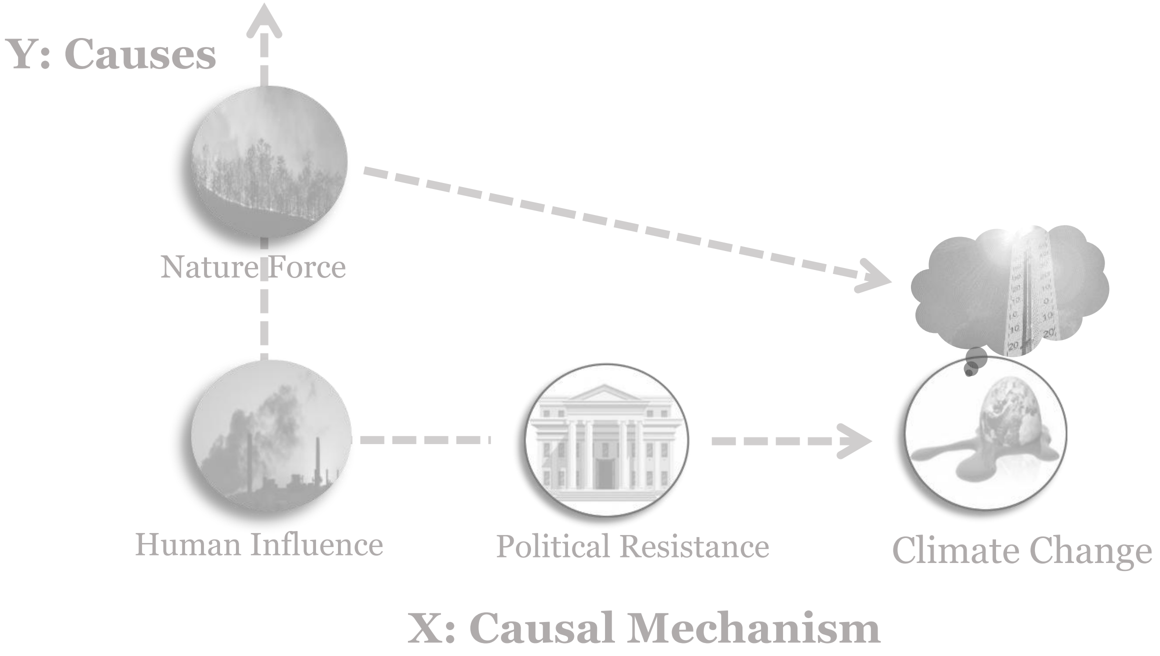
A **Causal Preemption** happened!

Analysis of Causes Based on Scenarios

Whether the Cause Can Sufficiently Sustain its Effect?



✕ Sustainable

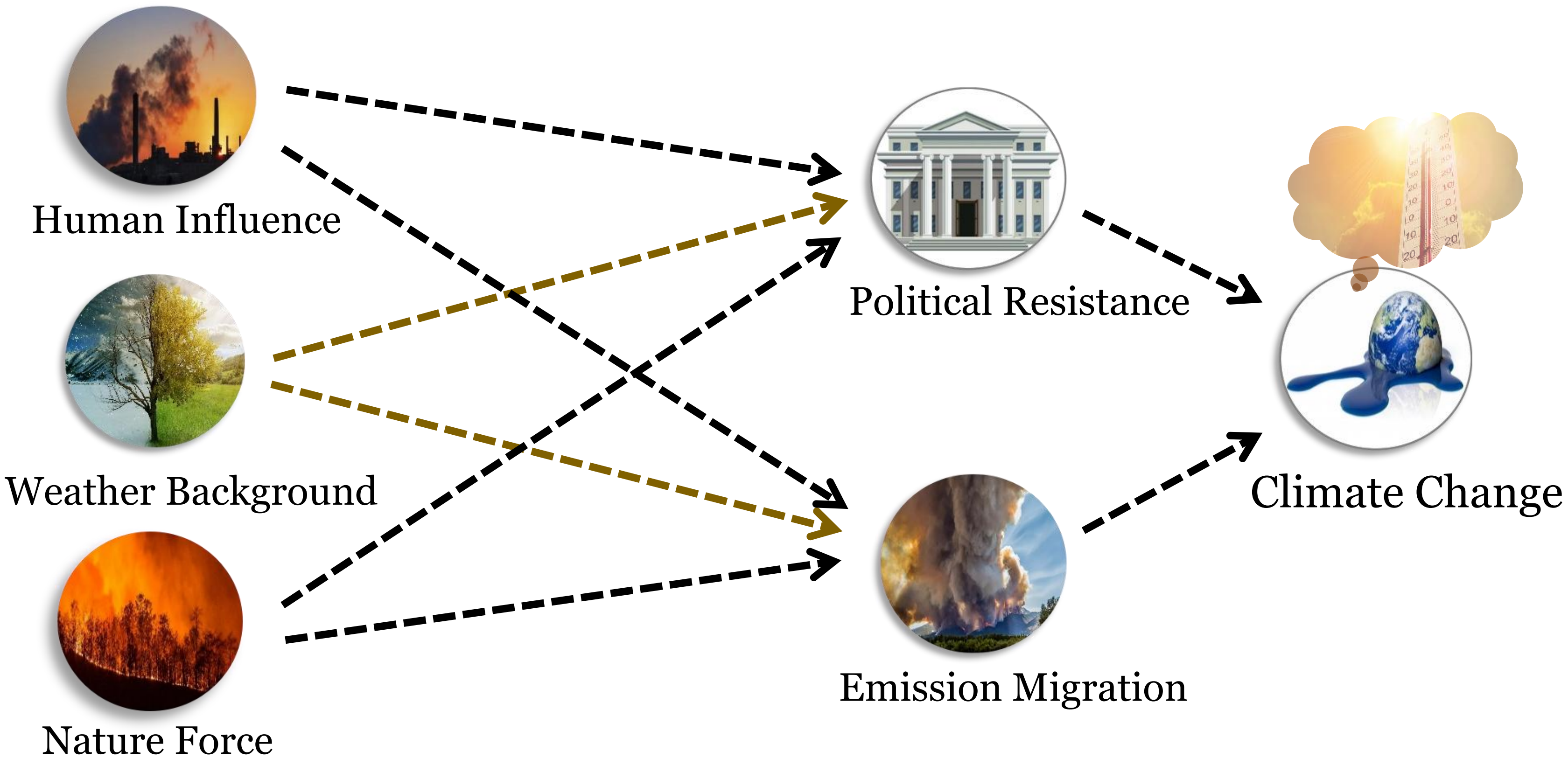


✓ Sustainable

Rule out some mechanisms in a causal diagram

To “slim down ” the causal diagram

A Causal Diagram of a General Scenario



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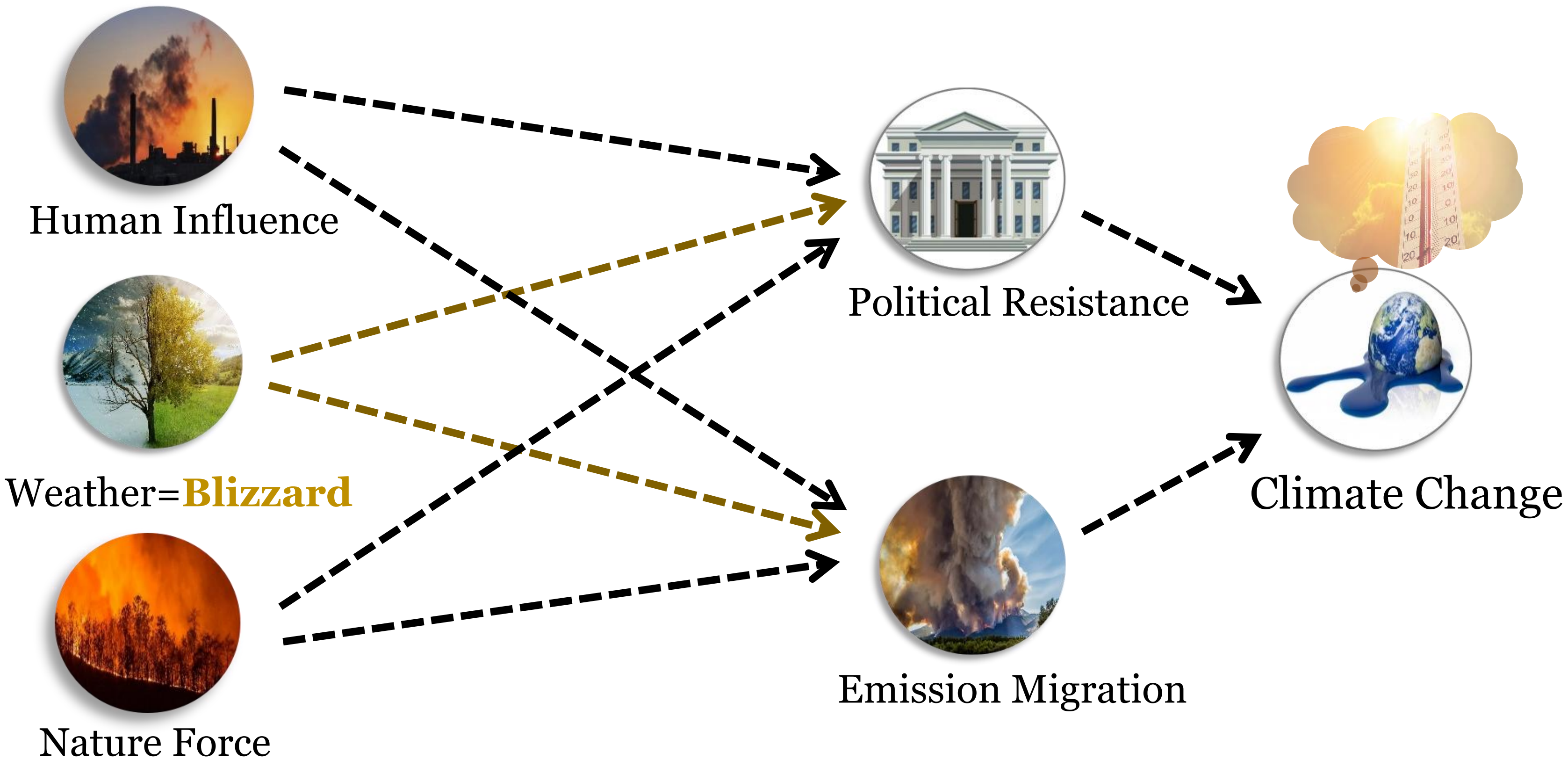
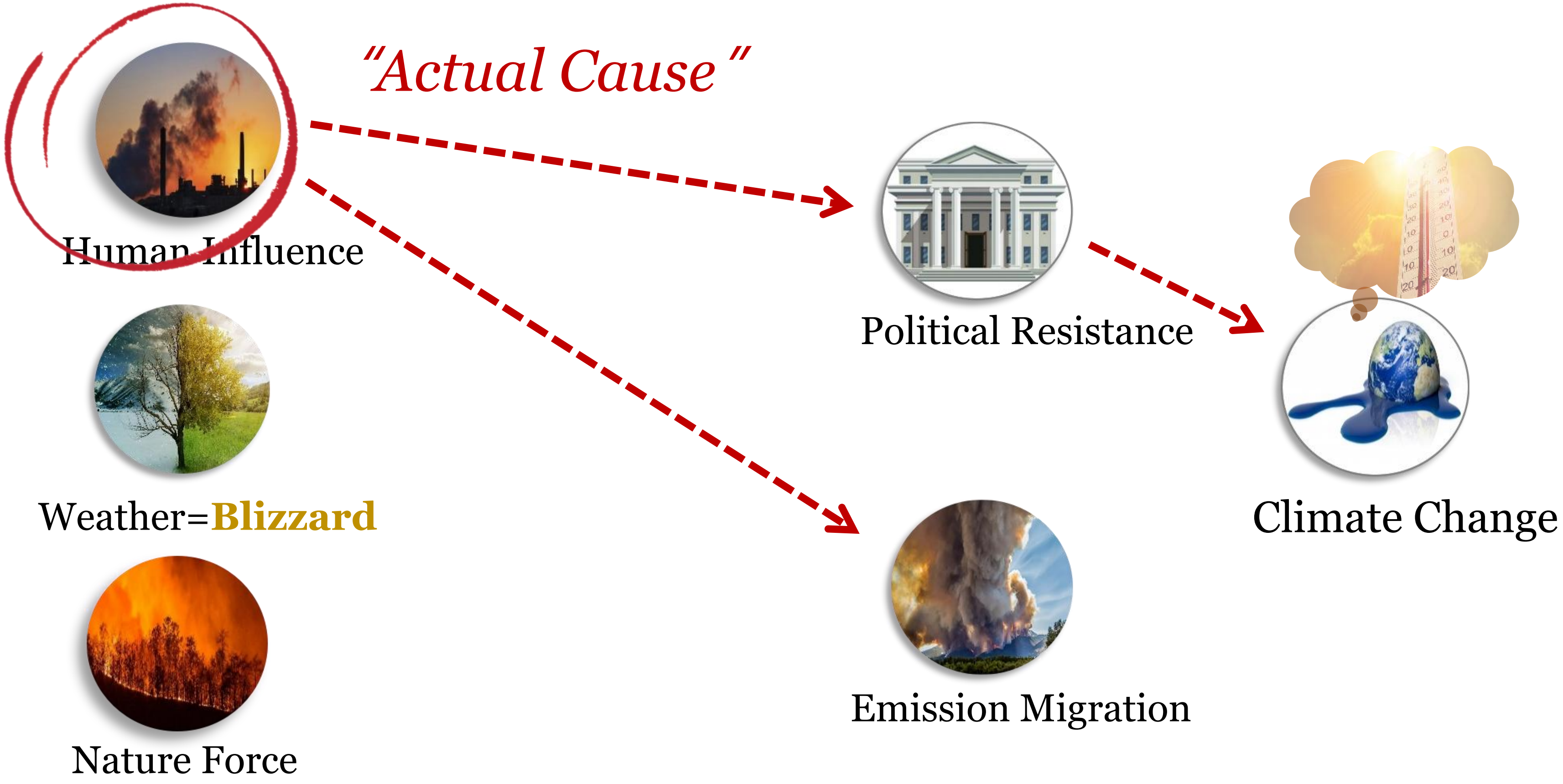


A **Causal Beam** of a **Specific** Scenario



*"It is **very likely** that **over half** the FAR (Fraction of Attribution Risk) of European summer temperature anomalies is **attributable to human influence**."*

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"Sufficiency and Necessary Analysis"

Insufficiency of Analysis of Causes ?

from Causal Diagrams to the Causal Beam

Sustenance-Based Causation

—Causality, Judea Pearl

REVOLUTIONARY CAUSATION

Can Vaccinal Treatments Save Lives of Loved Ones?

March 10. 2024

Xuanzhi Chen

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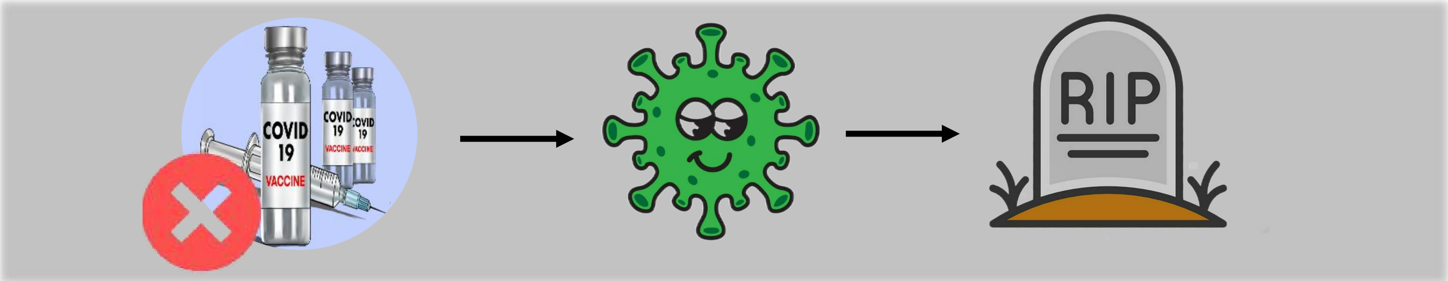
COVID-19 Has Taken away the Life of My Grandfather

My dear grandpa

Analysis of Causes:

***Why**? How could my grandpa had gotten an infection?*



*The **First Version** of the question "Why?"*

COVID-19 Has Taken away the Life of My Grandfather

My dear grandpa

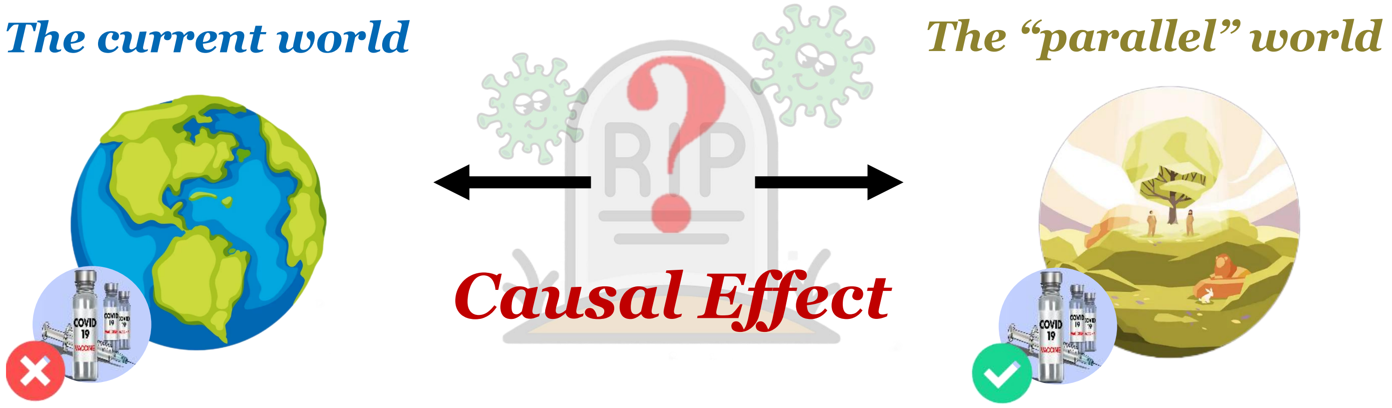
Inquiry About Causal Effect:

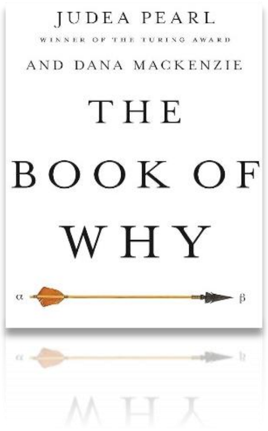
Why*? What is the mechanism by which vaccine prevent infection?*

The ***Second Version*** *of the question "Why?"*

Can Vaccinal Treatments Save Lives of Loved Ones?

> *"There is a* ***Second Version*** *of the 'WHY?' question, which we ask when we want to better understand the* ***connection*** *between a known cause and a known effect."*
>
> *—The Book of WHY, Judea Pearl*

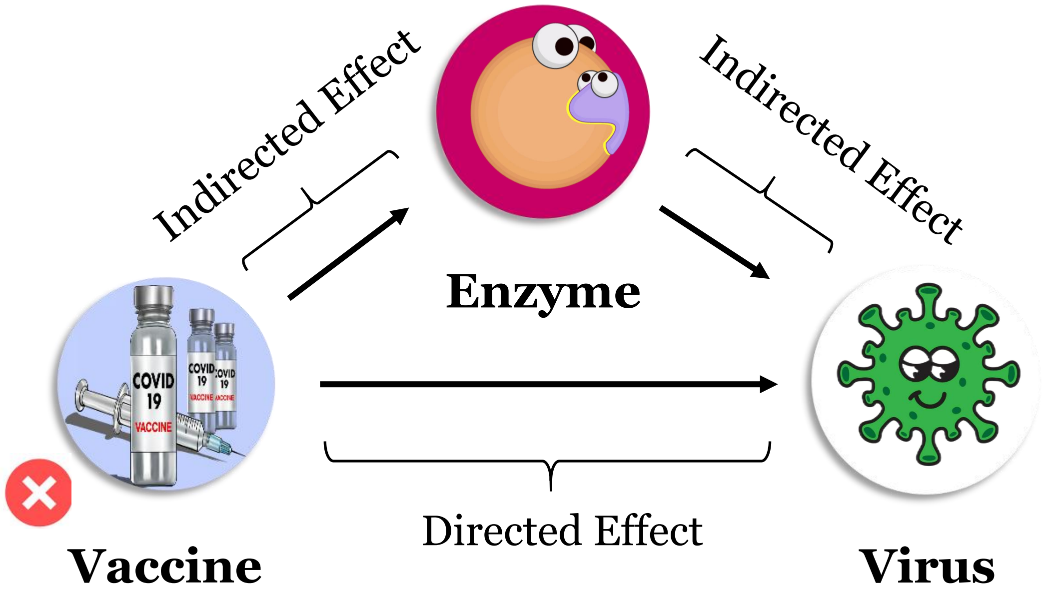




Inquiry About Causal Effect

- **Break Down the (Total) Causal Effect:**
  - Directed Causal Effect (the first version of WHY)
  - Indirected Causal Effect (the second version of WHY)

- **Directed Causal Effect (DE):**
  - Controlled Directed Effect (CDE)



*A plethora of **control** could sometimes lead to pitfalls!*

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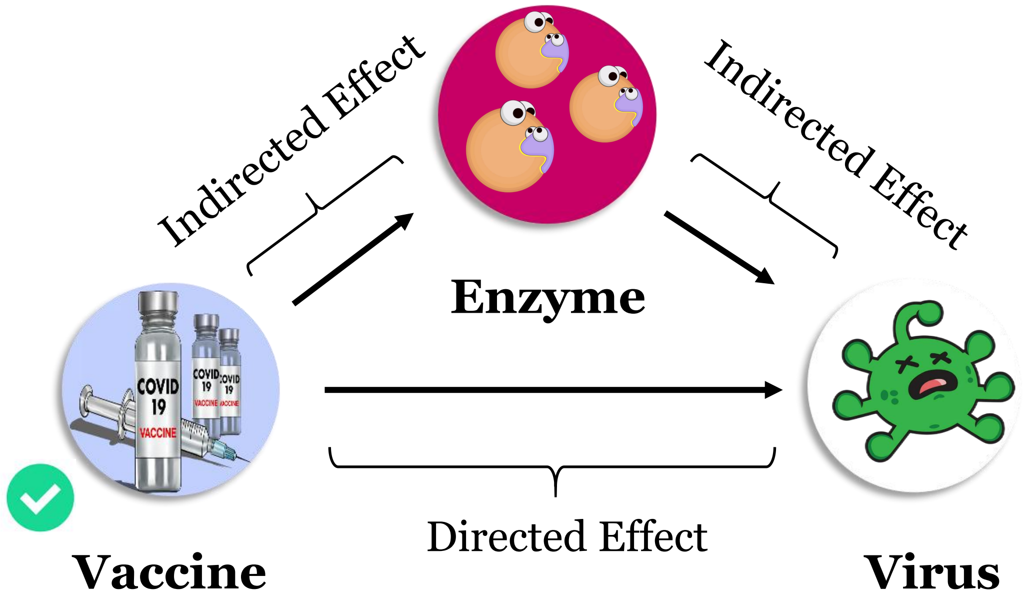

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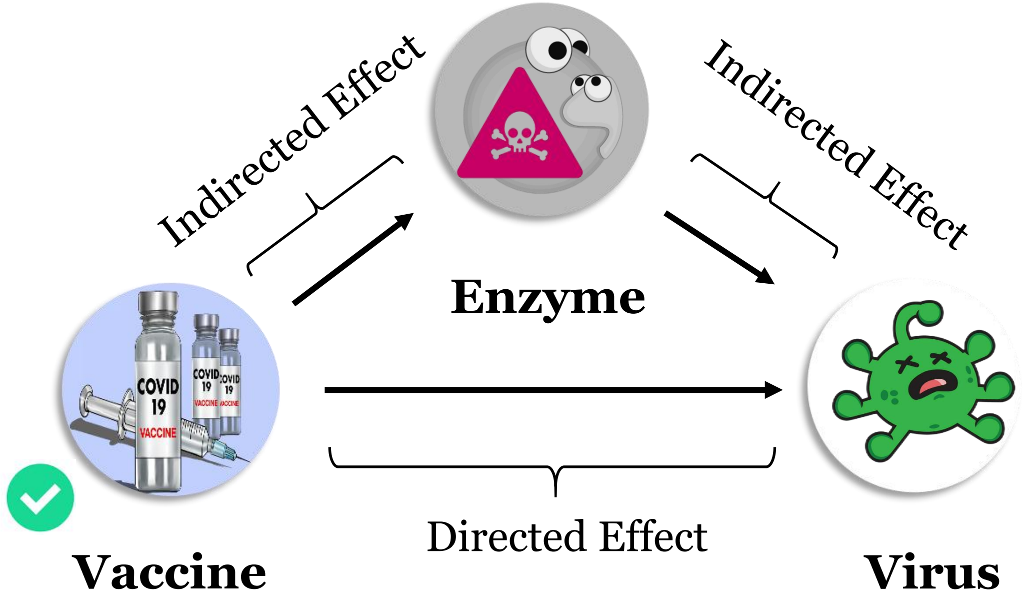

*A plethora of **control** could sometimes lead to pitfalls!*

- Break Down the (Total) Causal Effect:
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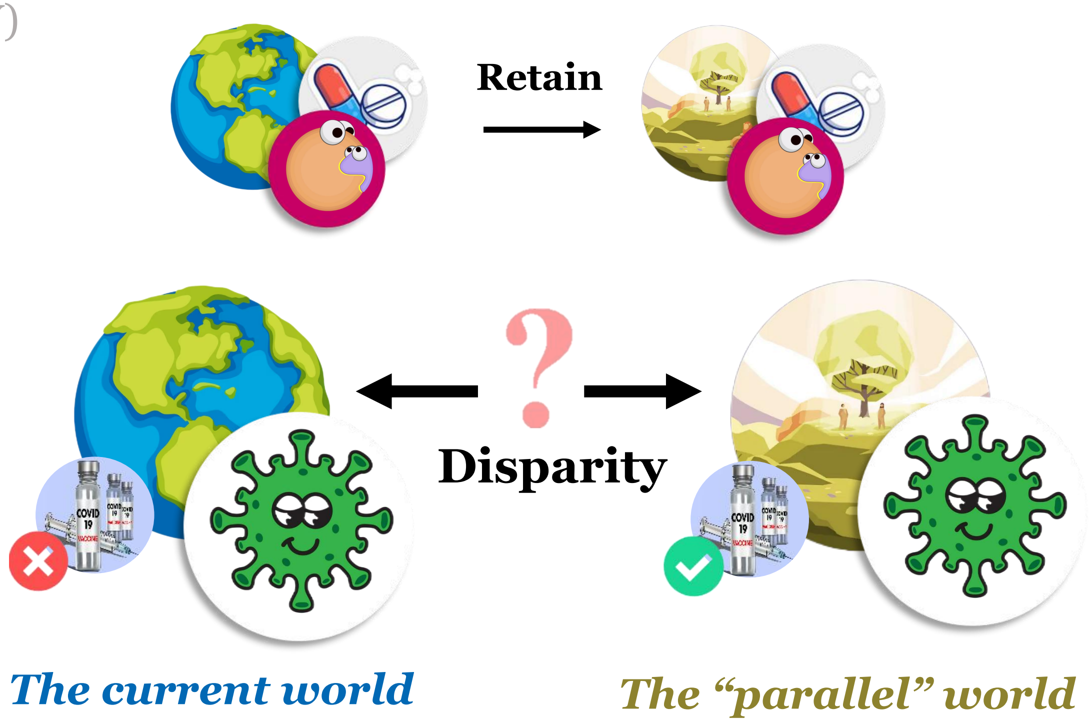
- **Directed Causal Effect (DE):**
  - Controlled Directed Effect (CDE)
  - Natural Directed Effect (NDE)

~~Control:~~ ***Hold constant the "preference":***



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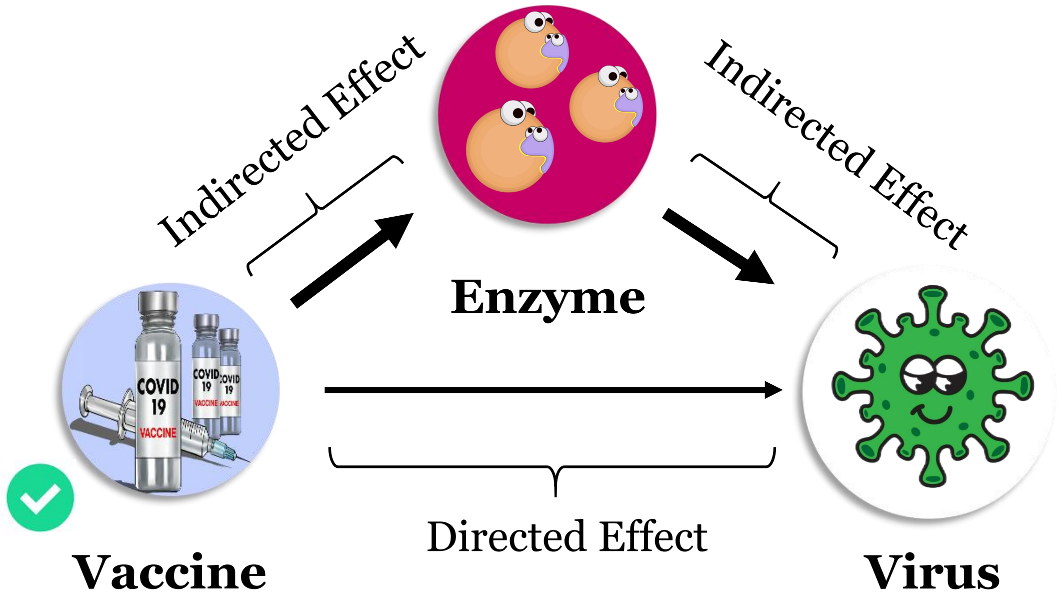

The (Natural) Directed Effect can be actually ***insignificant*** *!*

$$NDE = 0$$

Inquiry About Causal Effect

- **Break Down the (Total) Causal Effect:**
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- **Directed Causal Effect (DE):**
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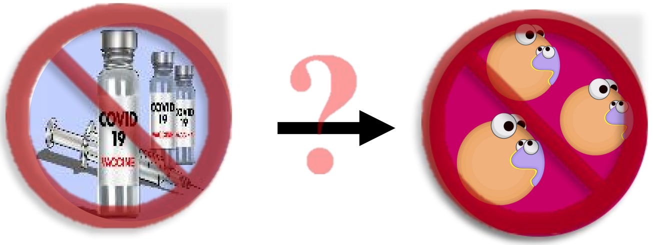
  - Natural Directed Effect (NDE)
- **Indirected Causal Effect (IE):**
  - ~~Controlled Indirected Effect (CIE)~~



*There is **No** Controlled Indirected Effect*

Inquiry About Causal Effect

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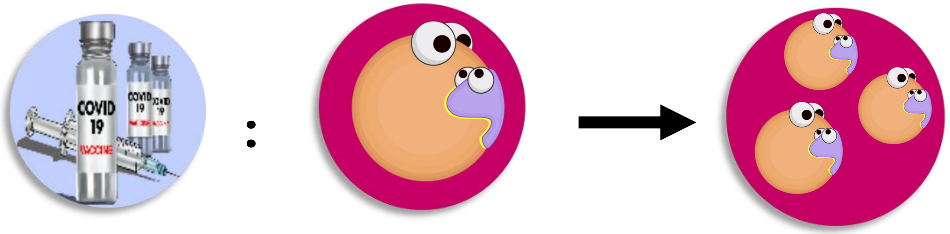
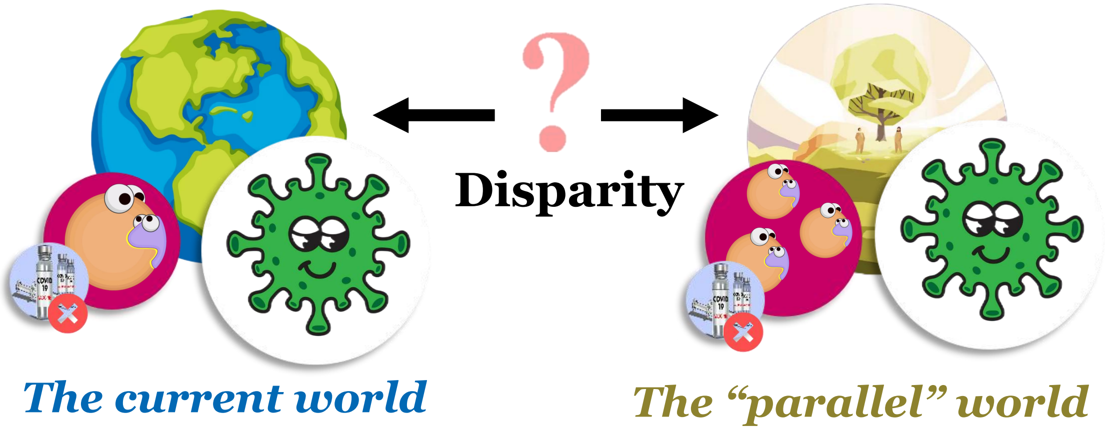
  - ~~Controlled Indirected Effect (CIE)~~
  - Natural Indirected Effect (NIE)





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  - ~~Controlled Indirected Effect (CIE)~~
  - Natural Indirected Effect (NIE)

"When I managed to strip the formula for the ***Natural Causal Effect*** *from all of its* ***Counterfactual Representation****, it was the greatest thrills in my life."*

—*The Book of WHY*, *Judea Pearl*

Inquiry About Causal Effect

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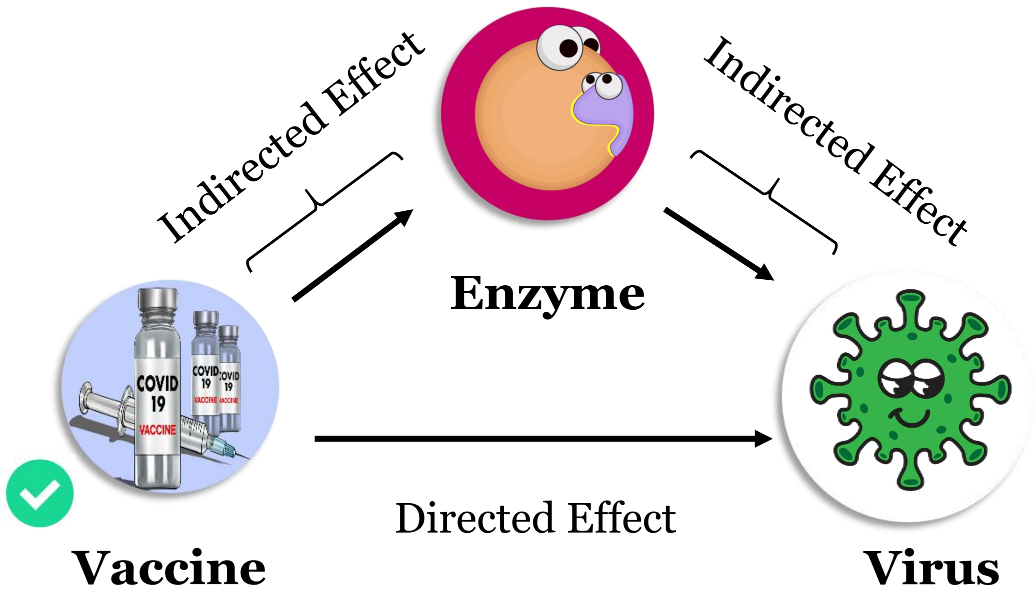

*The (Natural) Indirected Effect can also be **insignificant** !*

Inquiry About Causal Effect

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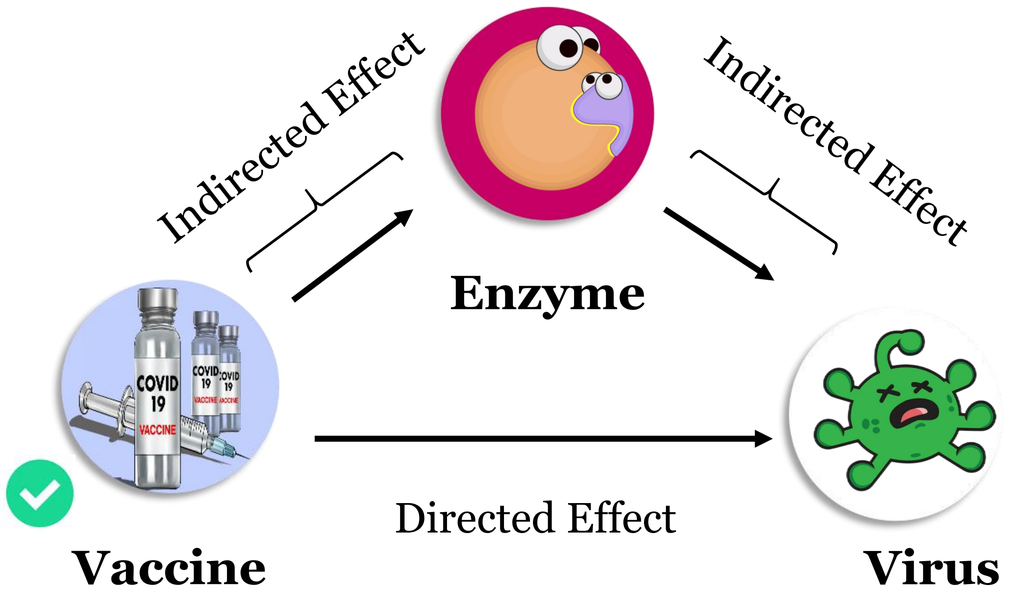
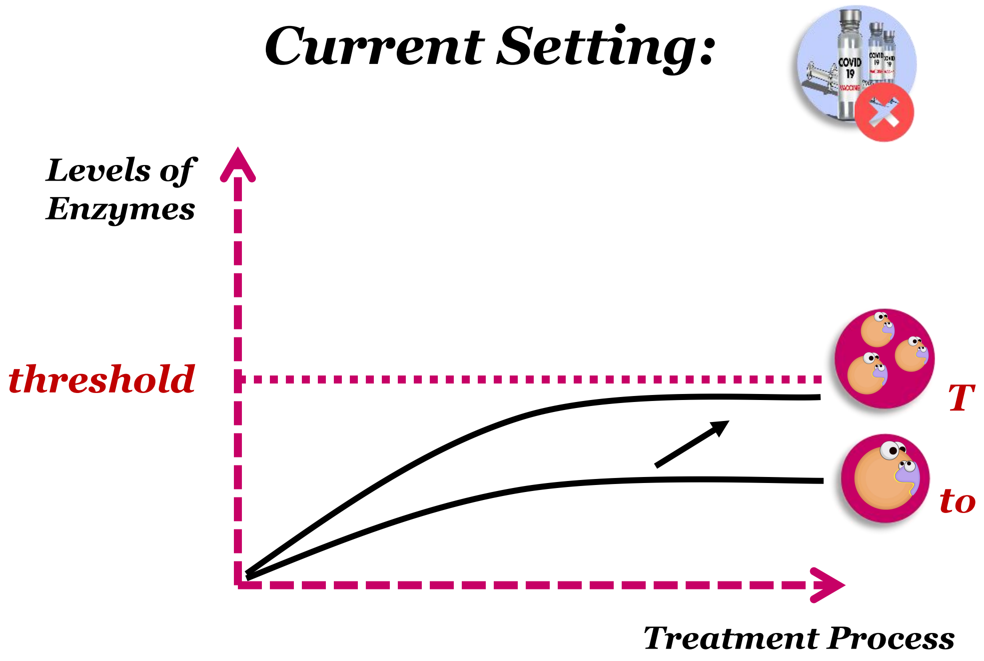


The (Natural) Indirected Effect can also be ***insignificant*** *!*

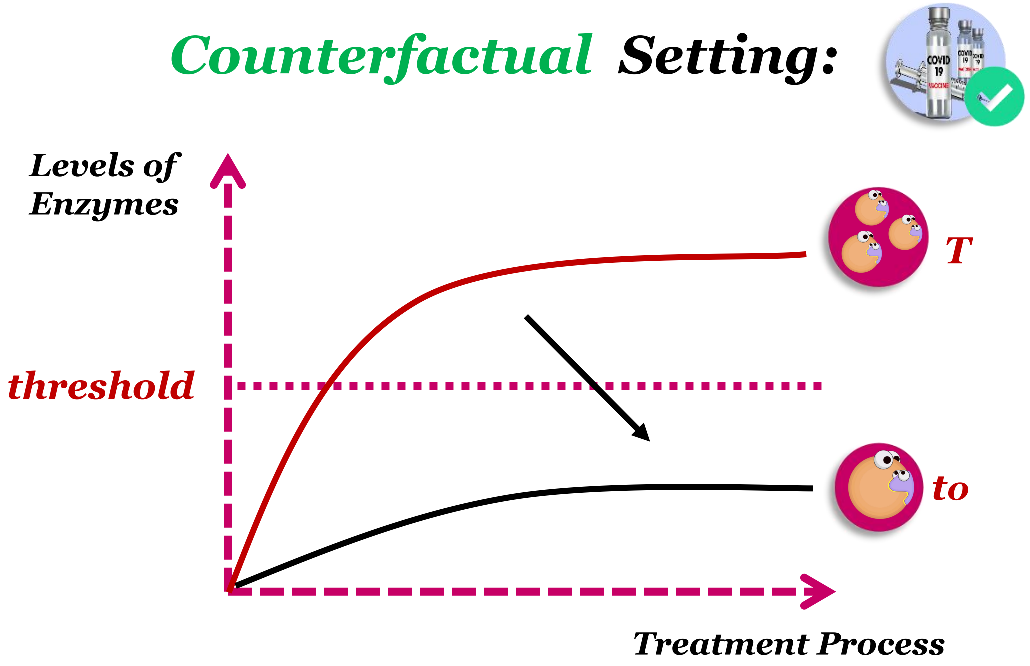
$$NDE = 0, NIE = 0$$

$$\boldsymbol{TCE = NDE + NIE = 0?}$$

Fix the Non-linearity Issue in Indirected Effects



$NIE = Y(T) - Y(t_0) = 0$

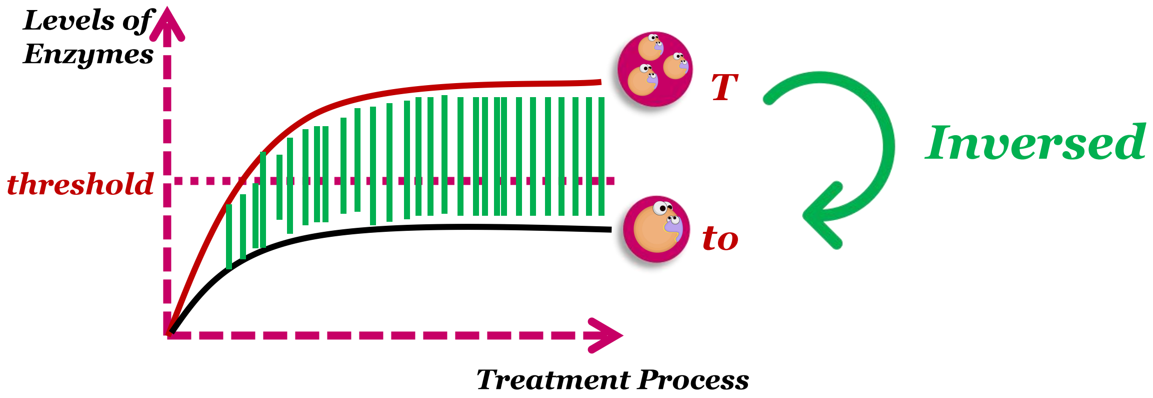


"Inversed" $NIE = Y(t_0) - Y(T) = -1$

Revelation of "Inversed" NIE (Natural Indirected Effect)

Total Causal Effect = Directed Effect + Indirected Effect. ***(Advantages)***

Inversed Indirected Effect: ***Potential Loss***



Revelation of "Inversed" NIE (Natural Indirected Effect)

Total Causal Effect = Directed Effect + Indirected Effect. ***(Positive Contribution)***

Inversed Indirected Effect: ***Potential Loss***

Total Causal Effect = Directed Effect - (Inversed Indirected Effect).

How to "Naturally" Calculate the Total Causal Effect?

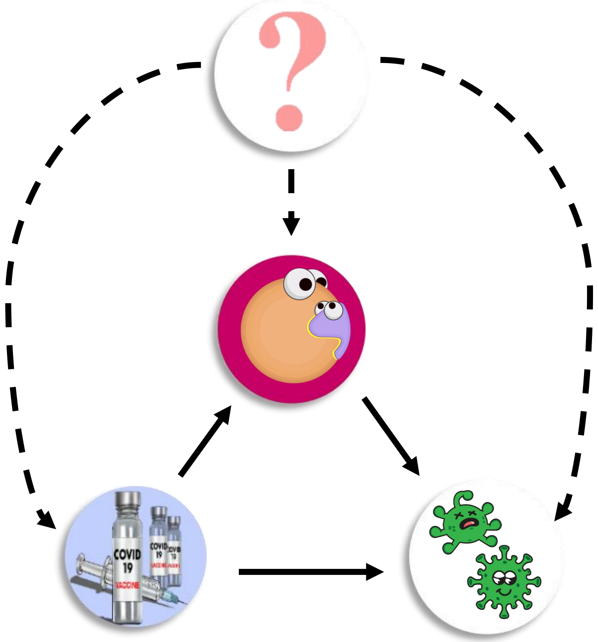
$$\text{Total Causal Effect} = \text{Directed Effect} - (\underbrace{\textbf{Inversed}\ \text{Indirected Effect}}_{\text{"Envisioning-Parallel-World"}}).$$

*"I was extremely thrilled to see this **subtraction principle** emerging from the analysis of **Total Causal Effect** (TCE), despite the **nonlinearity** of the equations."*

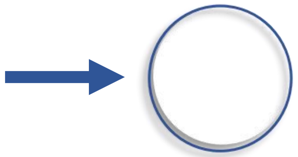
—The Book of WHY, Judea Pearl

*The magic as to the **subtraction principle** lies in: **Intervention** based on **Counterfactuals ideas***

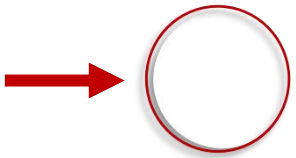
Execute Different Types of “Surgery" for Causal Diagrams



: ***Do*** *Operation*

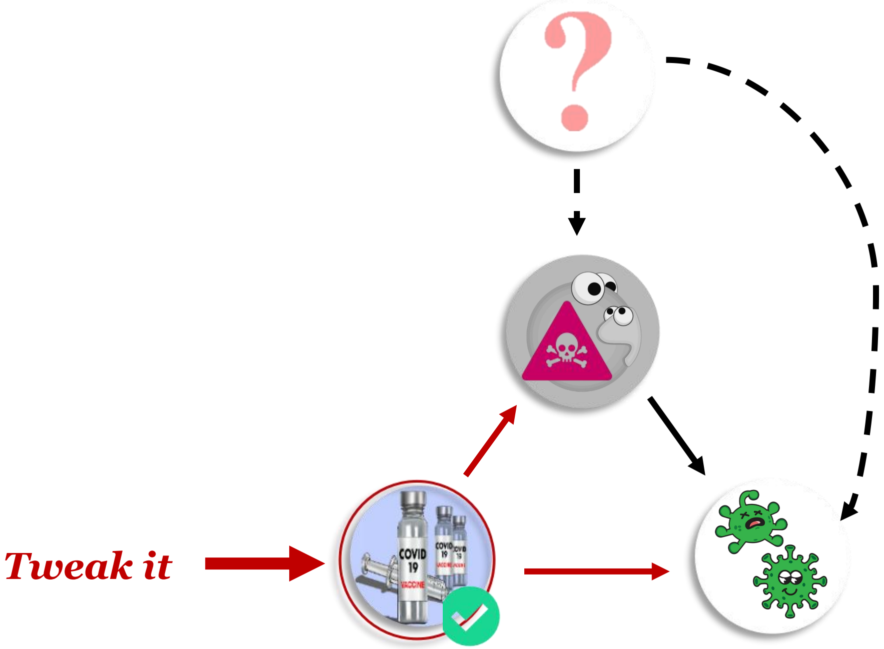


: Hold it Constant

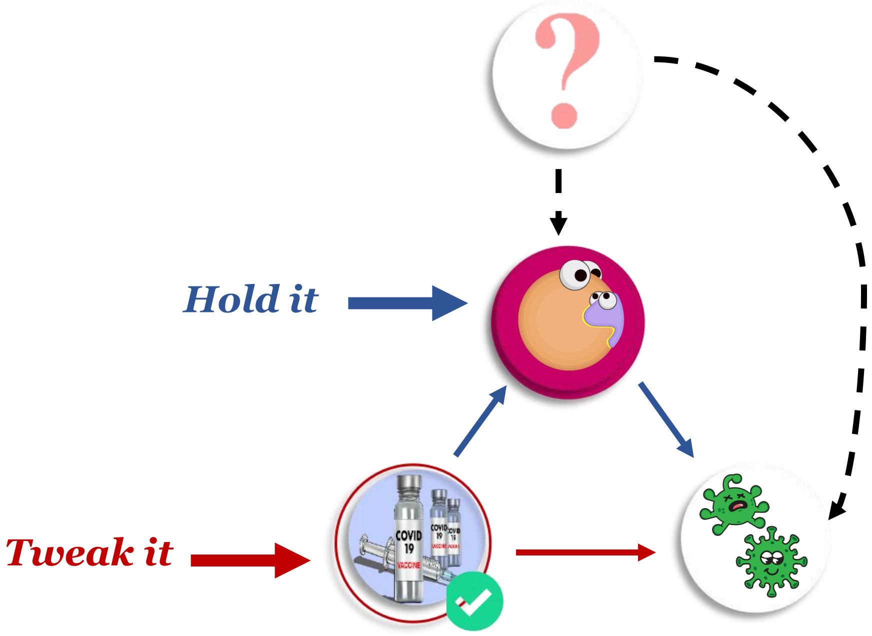


: Tweak it On Compulsion

Execute Different Types of "Surgery" for Causal Diagrams

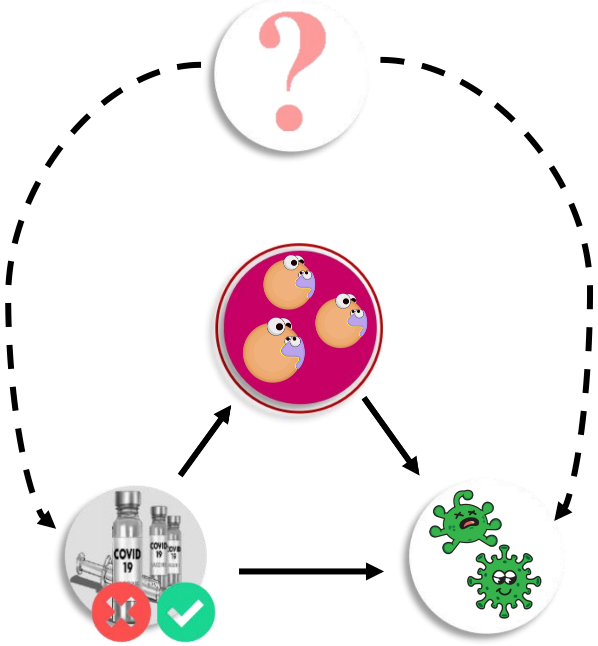


Controlled Directed Effect (CDE)

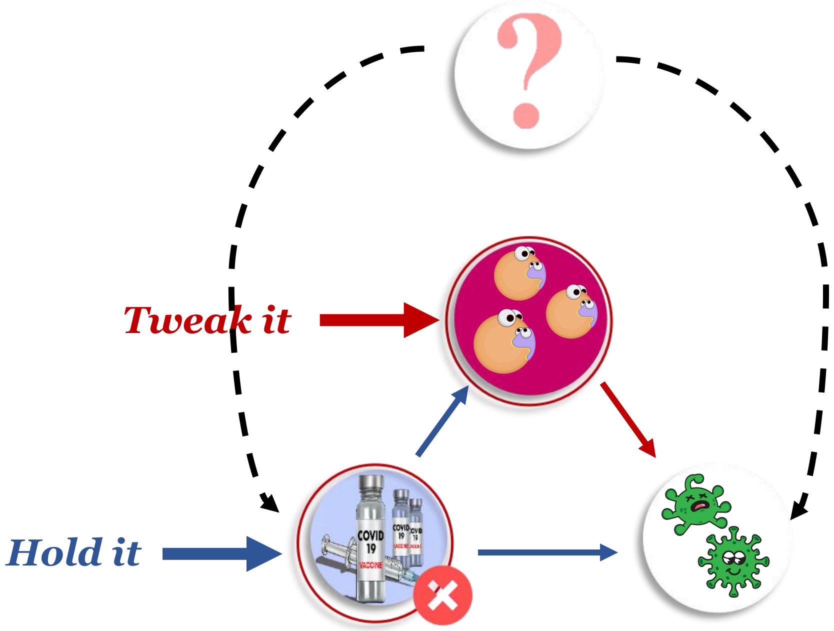


Natural Directed Effect (NDE)

Execute Different Types of “Surgery" for Causal Diagrams



~~Controlled Indirected Effect (CIE)~~



Natural Indirected Effect (NIE)

May my dear grandfather rest in peace in his ***Heaven World*****.**

愿我亲爱的爷爷（陈超伦）在他的天堂世界安息

INFERRED CAUSATION

Causal Diagram Learning = Data + Restriction

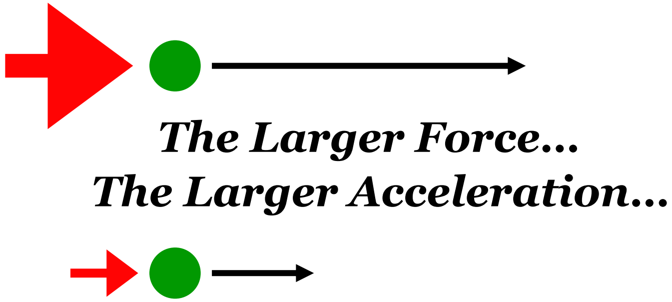
March 10. 2024

Xuanzhi Chen

Note: figures used in the presentation source Google Images Search; please contact if any infringement.

Why Does the Equation Harbor Causal Significance?

- **Causal Diagrams Entailed by Statistical Data**
  - Data-driven methods
  - Unstable probability distribution
  - Example: Newton's laws of motion



INVARIANT: F=ma

Why Does the Equation Harbor Causal Significance?

- **Causal Diagrams Entailed by Statistical Data**
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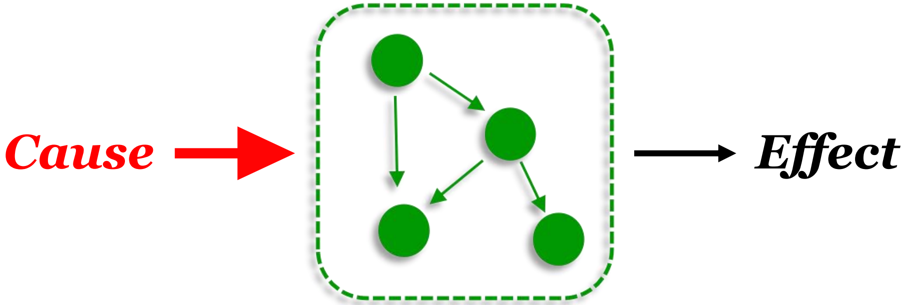
  - Akin to the “Law of Causality”
  - The cause is irrelevant to the causal mechanism



INVARIANT:

“Law of Causality”

Why Does the Equation Harbor Causal Significance?

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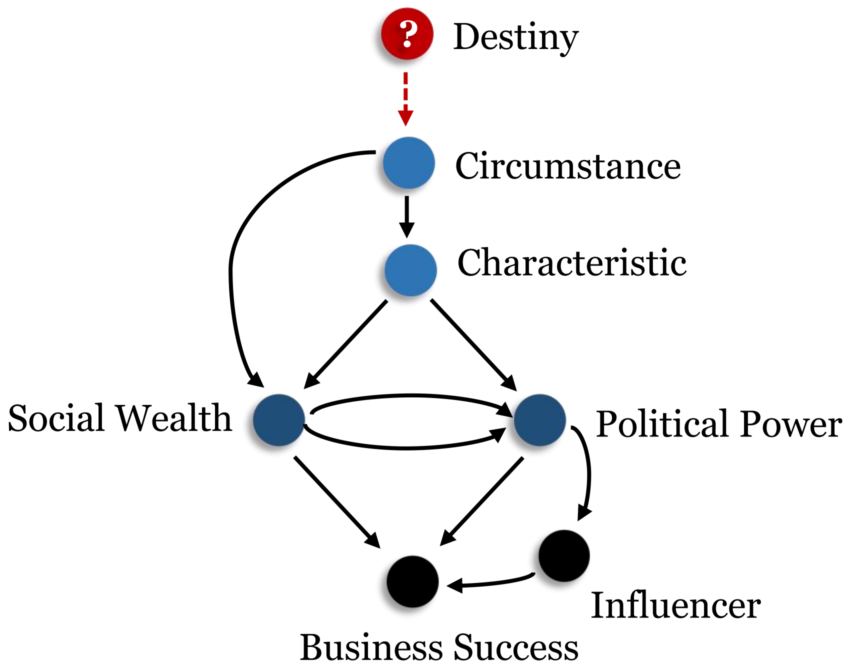
- **Invariant Mechanisms Embedded in Causal Structures**
  - Recall the Example: a Diagram of our Lifetime Events
  - Moral of an arrow: "Autonomous" mechanism



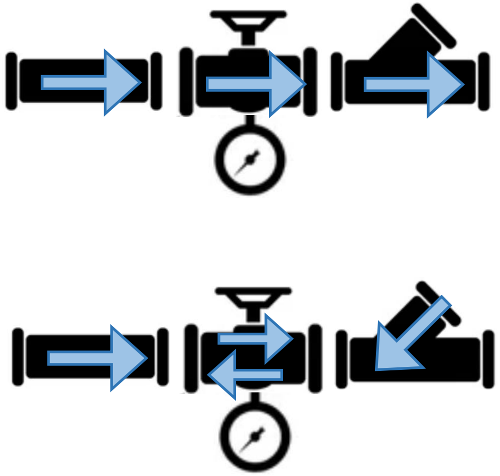
Why Does the Equation Harbor Causal Significance?

*"If we accept that causation **cannot** be inferred from **statistics** alone, then the **Markovian** equivalent models is inevitable"*

—Judea Pearl, Causality

Markovian Property: Conditional Independence

“Conditional independence is the heart in causal modelling” —Judea Pearl



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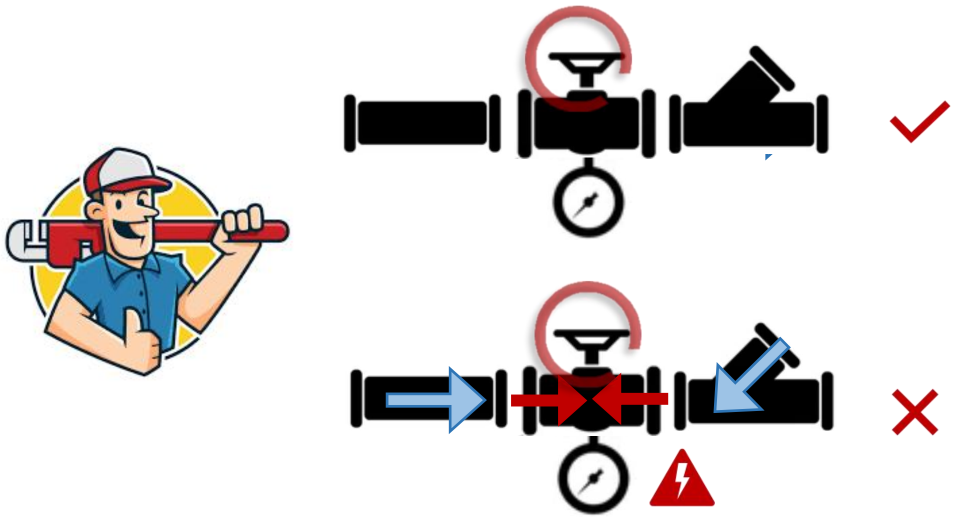

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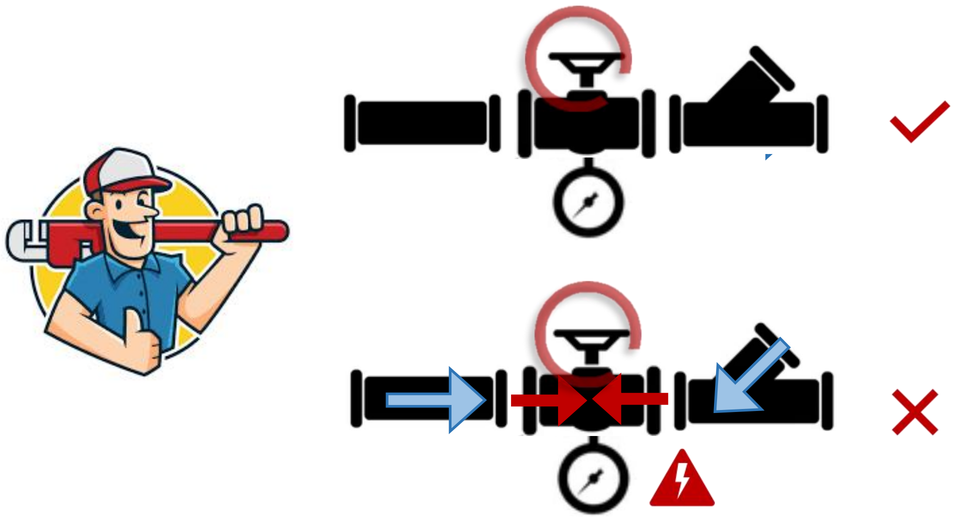

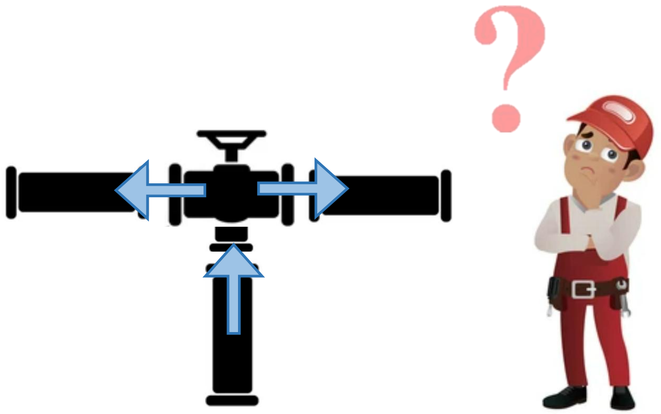

Think of the flow of water carried by a pipe as:

the flow of causal-dependence carried by an arrow

Markovian Property: “Deterministic” Systems

- **Working States of Computer, Keyboard, and Mouse:**
  - ① The keyboard and mouse work when the button on ✓
  - ② The keyboard and mouse are stilled when the button off ✓
  - ③ The keyboard and mouse are (keeping) stilled when the button on ✗

Markovian Property: "Deterministic" Systems

- **Working States of Computer, Keyboard, and Mouse:**
  ① The keyboard and mouse work when the button on ✓
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- **"Causal Markov " Doesn't Hold in "Indeterministic Systems"**

Markovian Property: “Deterministic” Systems

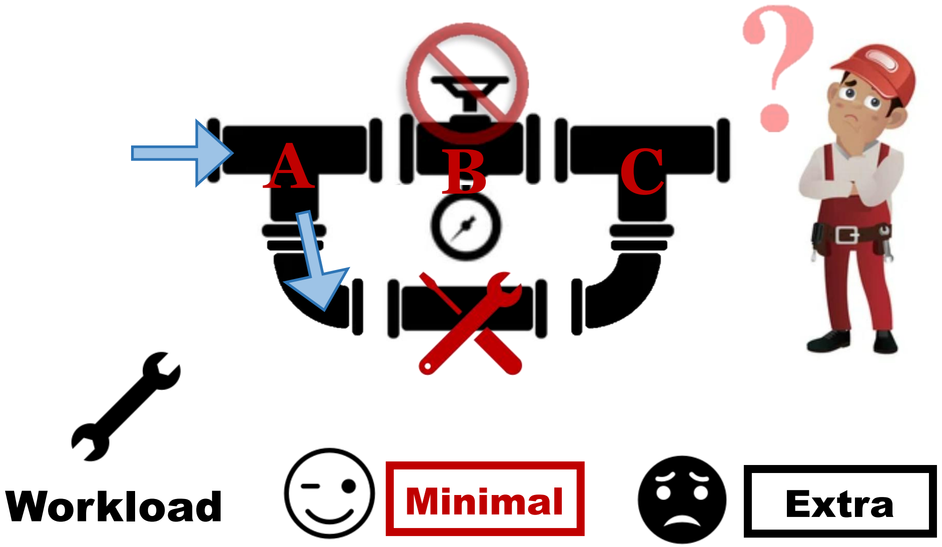
- **Working States of Laptop, Keyboard, and Mouse:**
  - ① The keyboard and mouse work when the button on ✓
  - ② The keyboard and mouse are stilled when the button off ✓
  - ③ The keyboard and mouse are (keeping) stilled when the button on ✕
- **“Causal Markov ” Doesn’t Hold in “Indeterministic Systems”**
- **Good News: It Will Hold Again in a “Microscopic" System**

Causal Minimality and Causal Faithfulness

"Aside from independence, what about dependence?"

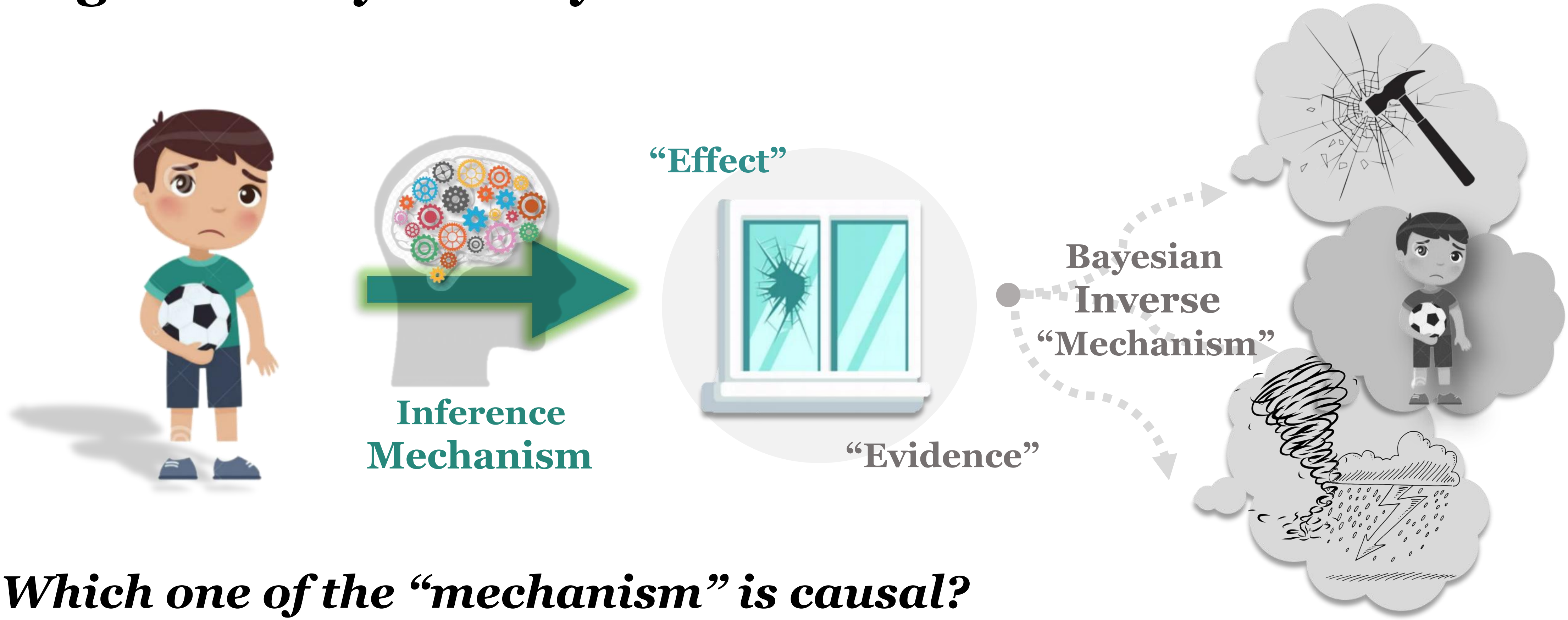
"Restriction **prohibiting** the dependence of **causal insignificance**"

*"to work **normally** "*



Restrictions Give Rise to Identification

Cognition Asymmetry

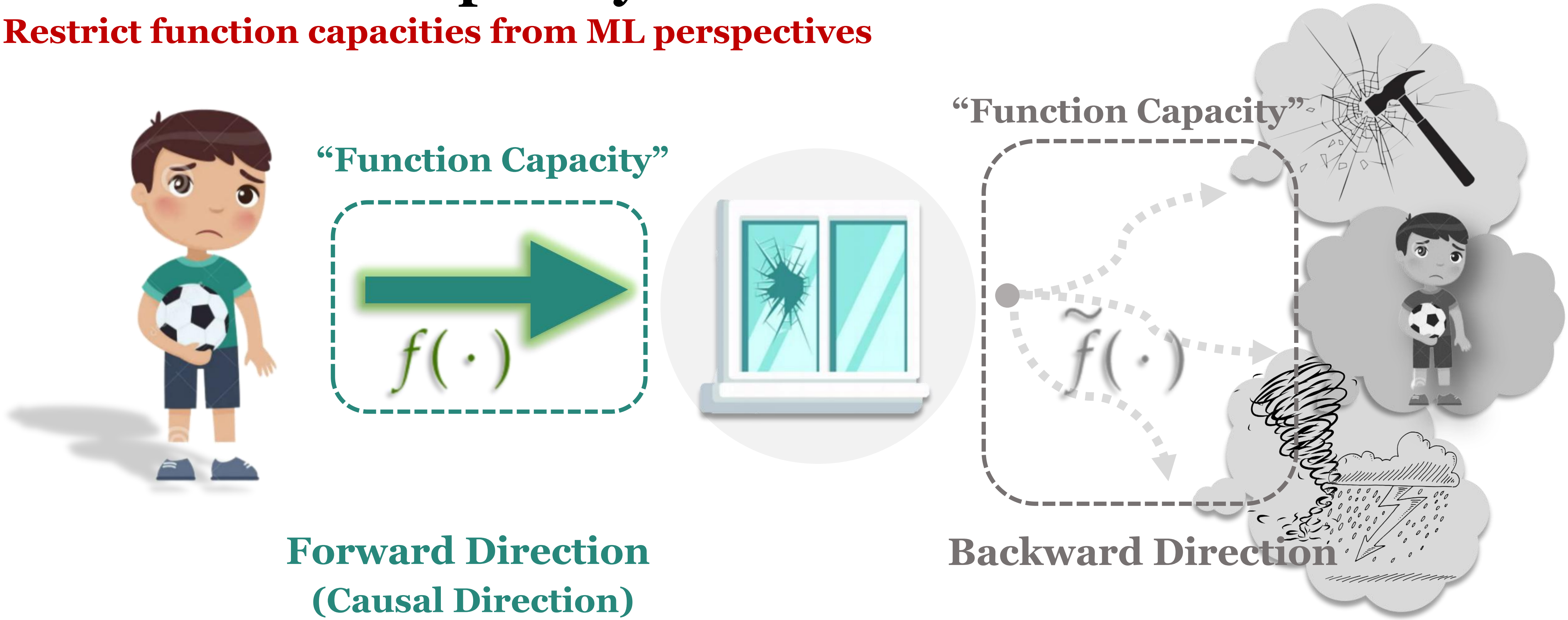


Which one of the "mechanism" is causal?

Restrictions Give Rise to Identification

Mechanism Complexity

Restrict function capacities from ML perspectives



INFERRED CAUSATION

Challenges: Unmeasured Common Cause

March 10. 2024

Xuanzhi Chen

Note: figures used in the presentation source Google Images Search; please contact if any infringement.

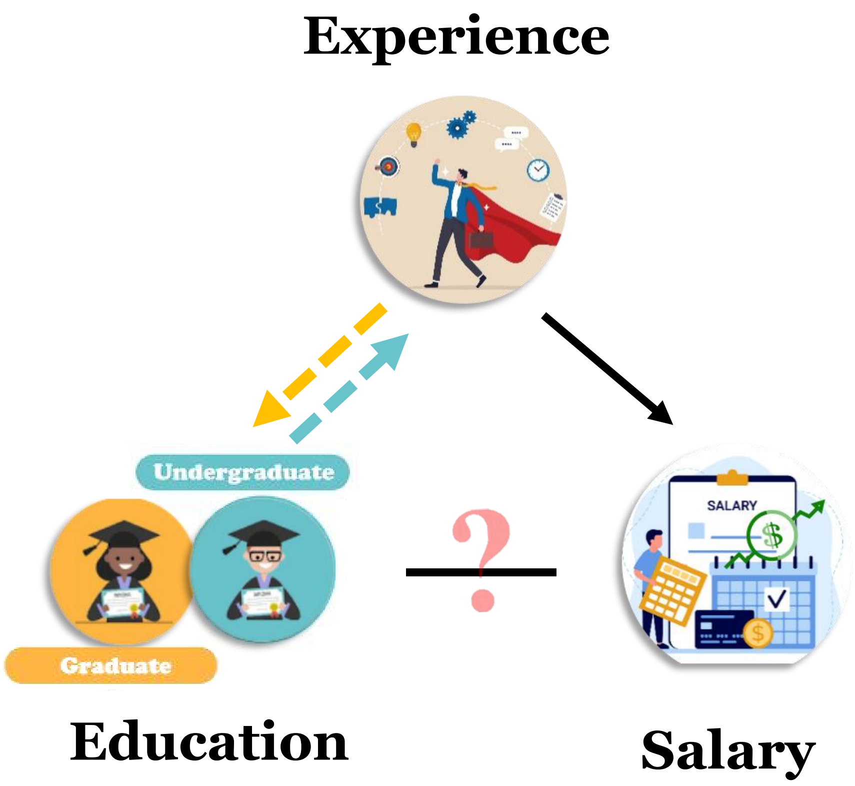
Causal Diagram's Philosophy

Horizontal: Perception is Important than Operation

Vertical: Elaboration of High-Level Causal Semantics

➢ Example from The Book of WHY

- ➢ Education, salary, experience
- ➢ Controlled experiments
- ➢ Be care of the possible mechanism



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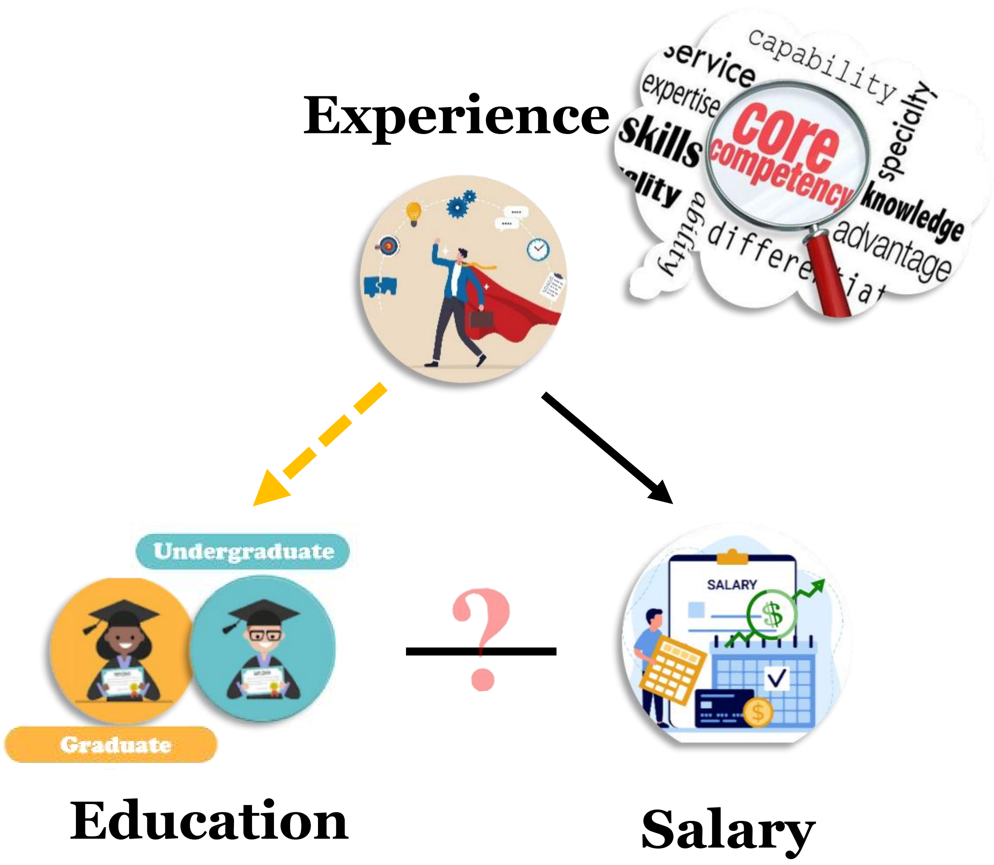

Challenges: Unmeasured Common Cause

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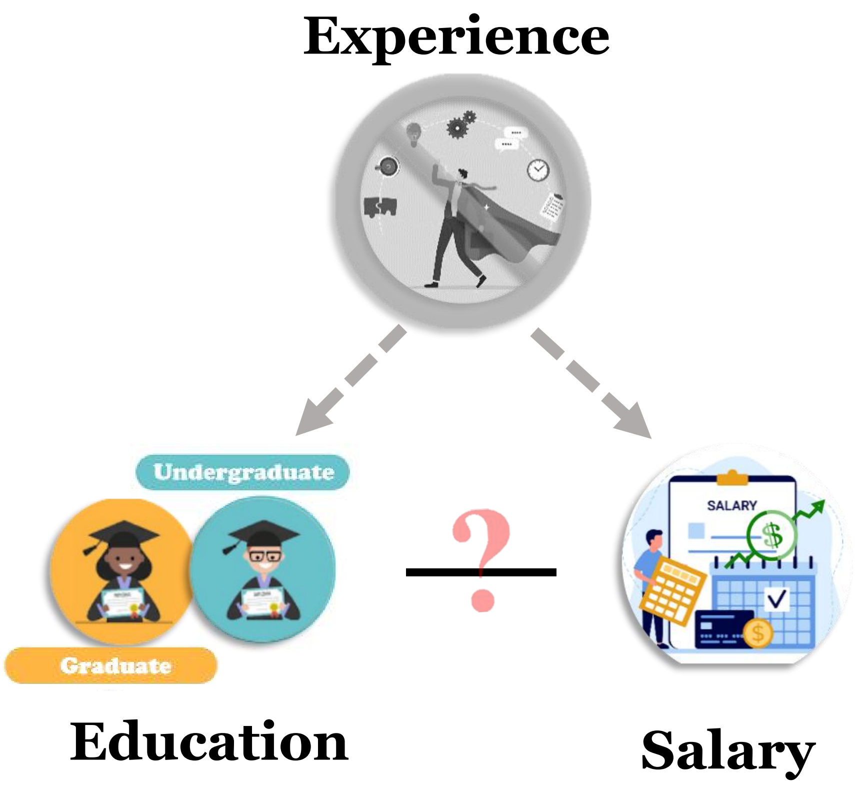

Challenges: Unmeasured Common Cause

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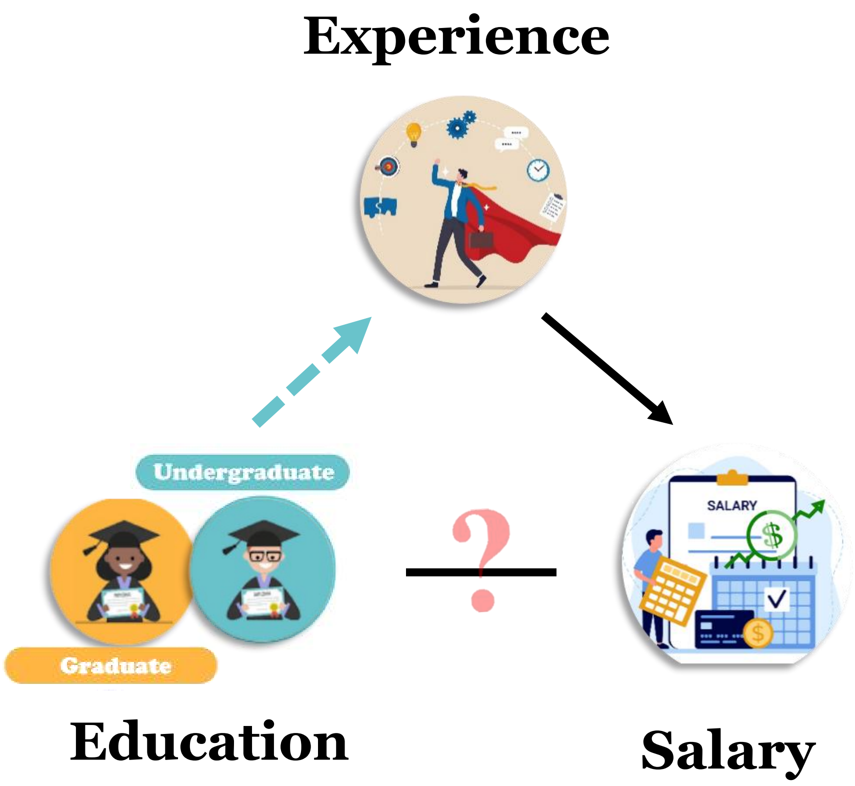

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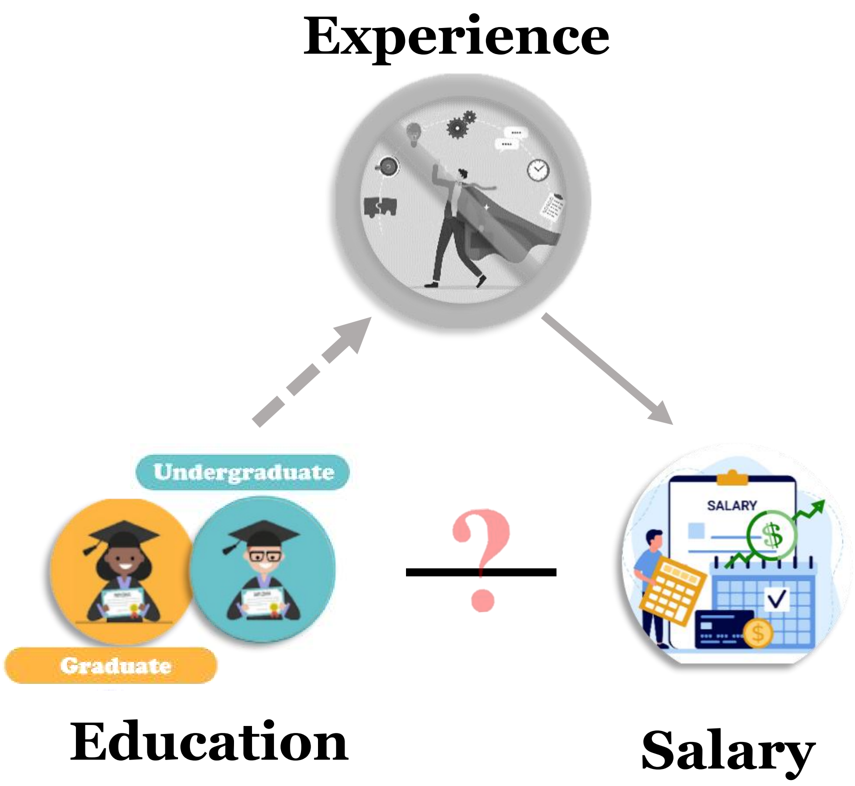

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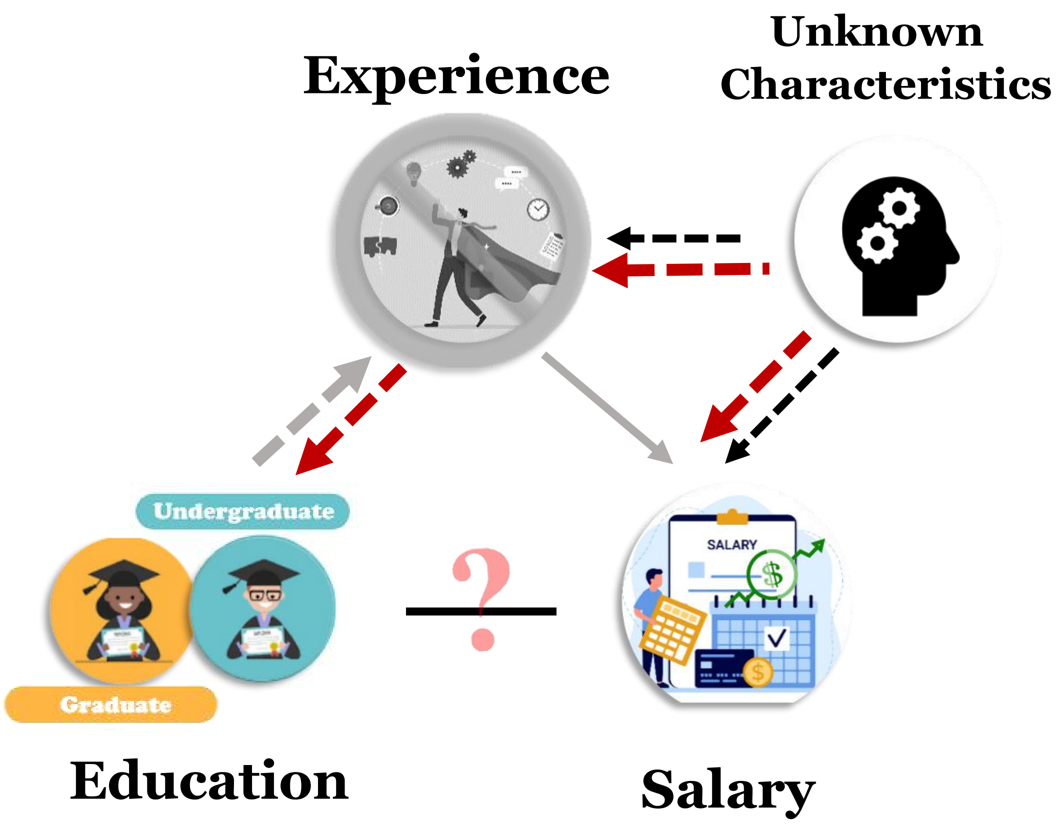

Challenges: Unmeasured Common Cause

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- **Example from The Book of WHY**
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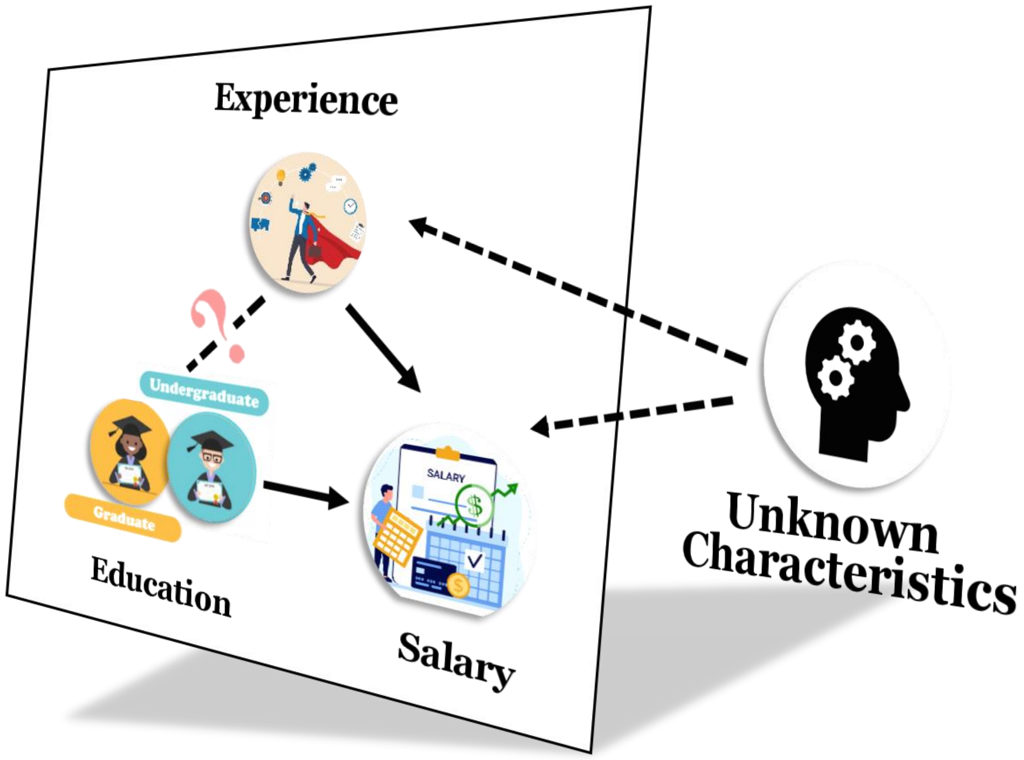
- **Education -> Experience**
- **Control it or Keep Unmeasured?**



Causal Diagram's Philosophy

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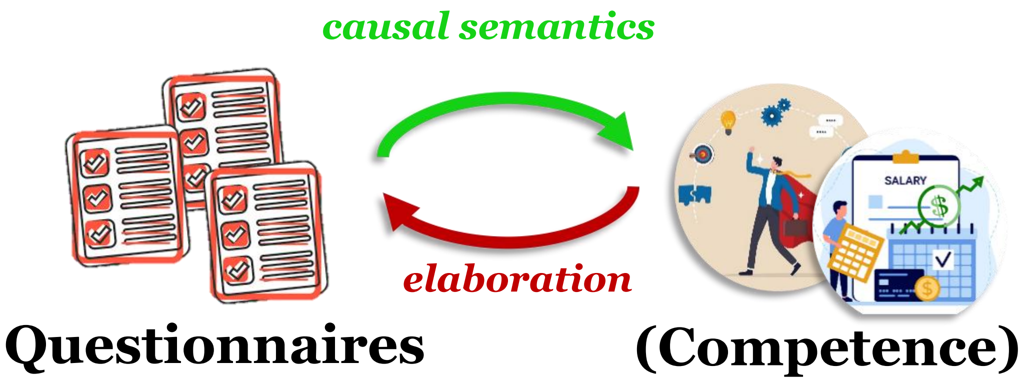
- **Questionnaires to Characterize the Factors**
  - Superior factors for causal semantics
  - **Elaboration**: measurement model



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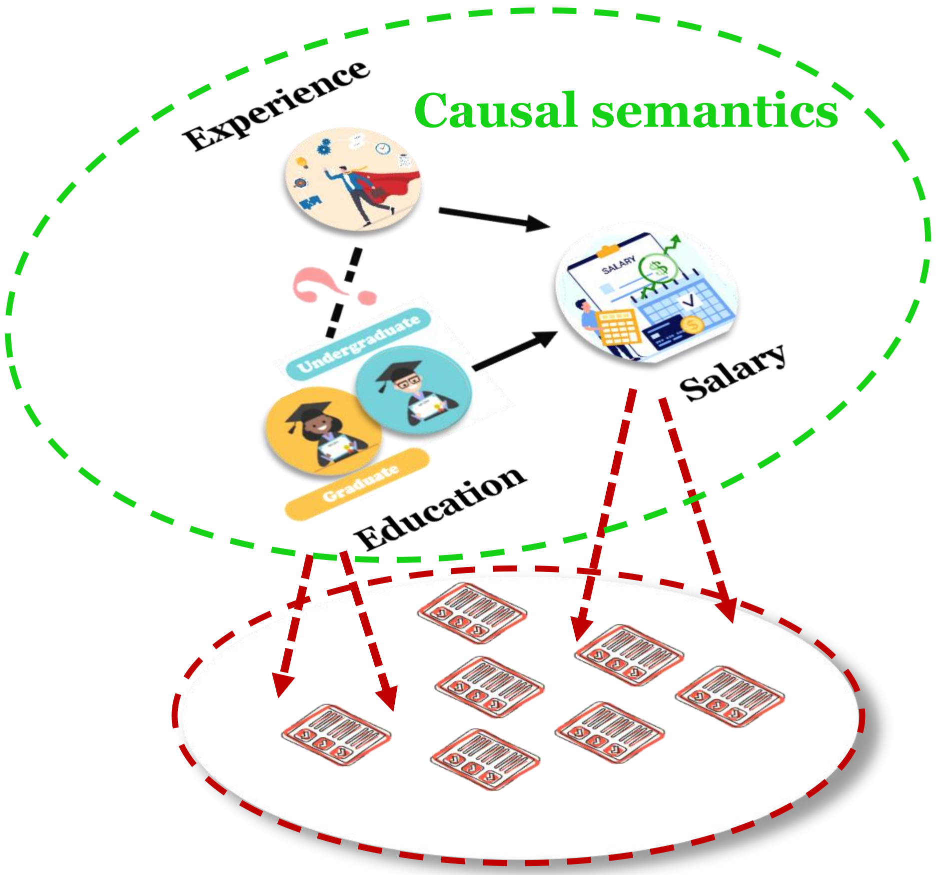


Challenges: Unmeasured Common Cause

Concept of Inducing Path

“Correlation is not causation.”

Correlation - **(Conditional) Independence** = **(Causal) Dependence**

“Correlation > Causation.”

Concept of Inducing Path

"Correlation is not causation."

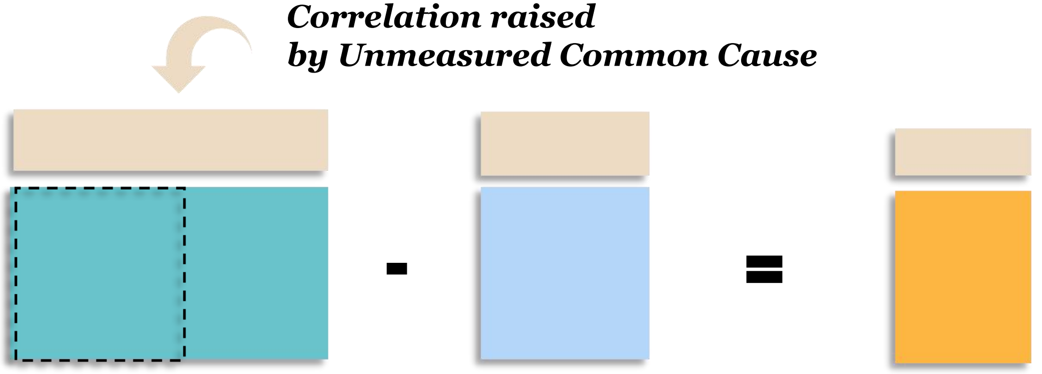
Correlation ***(Conditional) Independence*** ***Causal Path***

"Correlation > Causation."

Concept of Inducing Path

"Correlation is not causation."

Correlation raised by Unmeasured Common Cause



Correlation

(Conditional) Independence

~~(Causal)~~ Dependence

"Correlation > Causation."

Concept of Inducing Path

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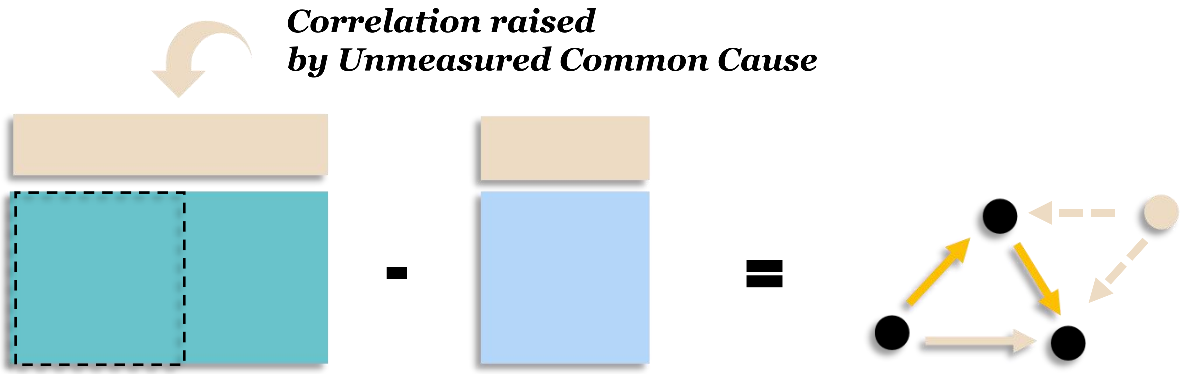


Correlation

(Conditional) Independence

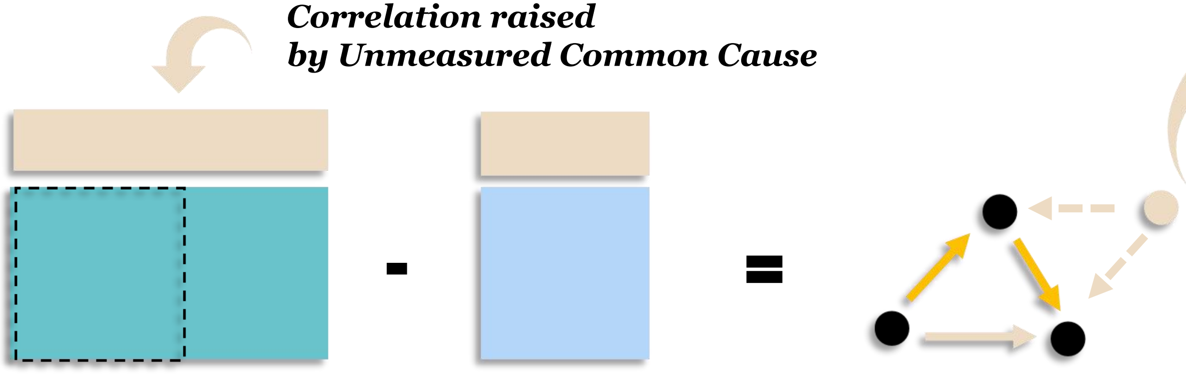
~~(Causal)~~ **Dependence**

“Correlation > Causation.”

Concept of Inducing Path

“Correlation is not causation.”

Correlation raised
by Unmeasured Common Cause



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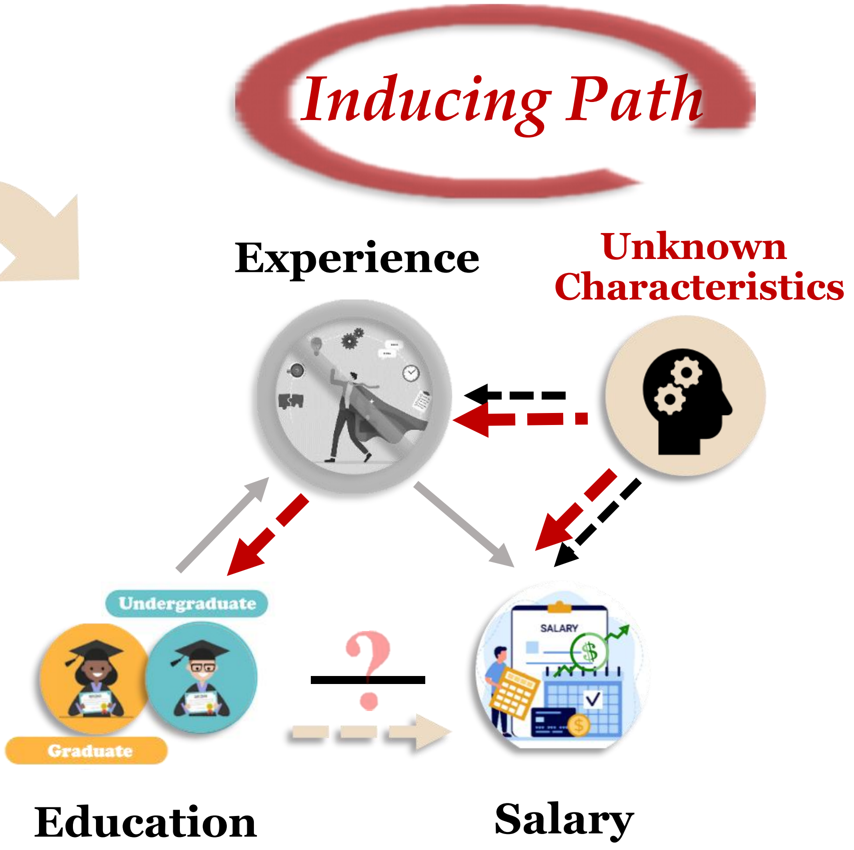
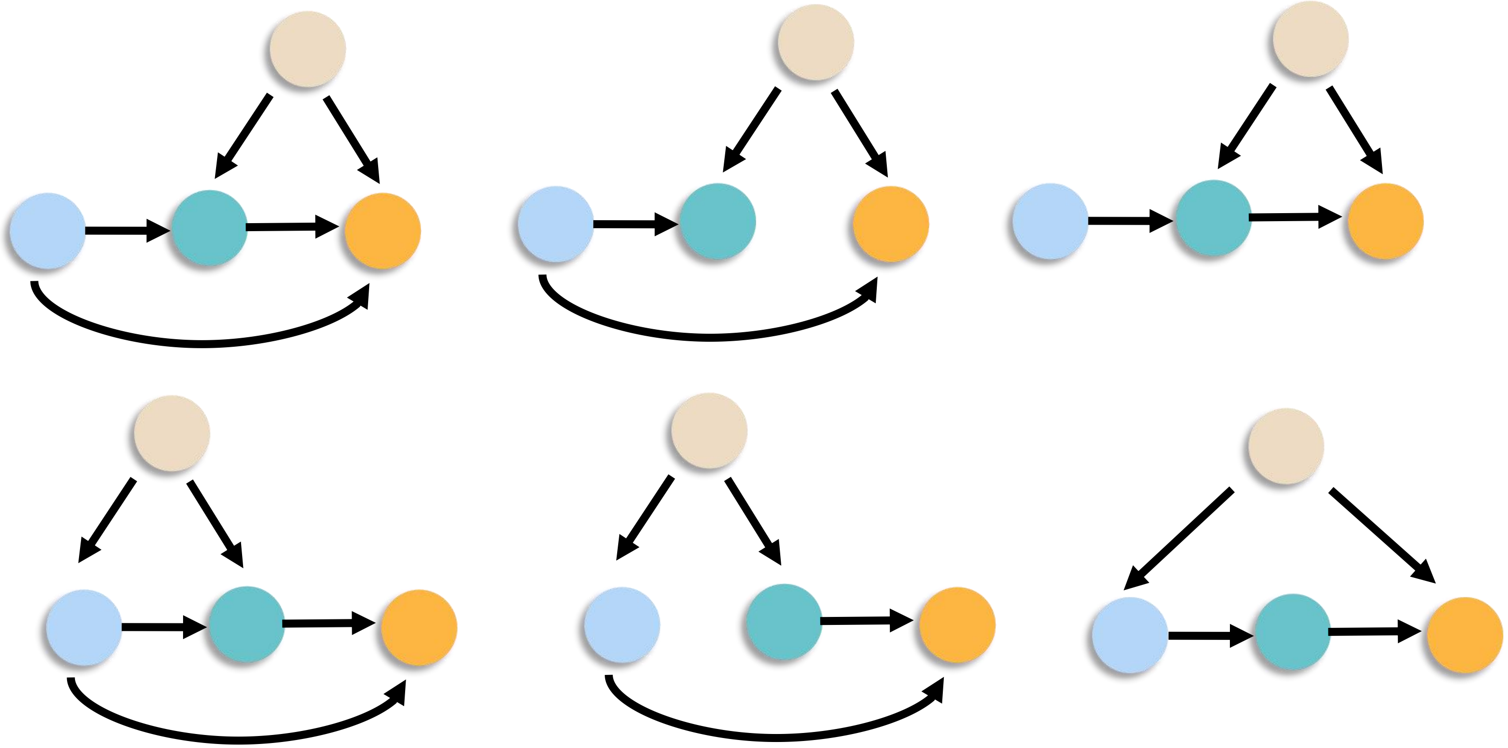
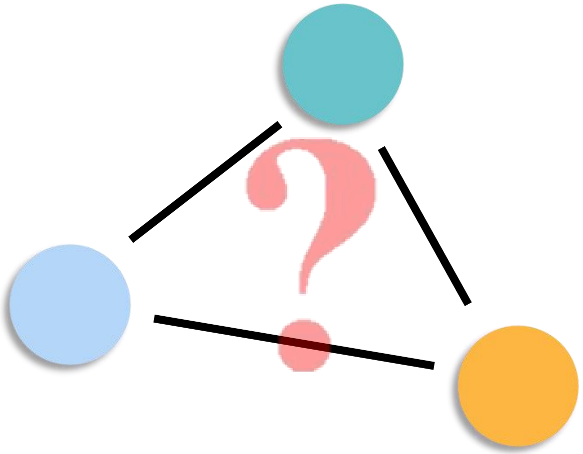


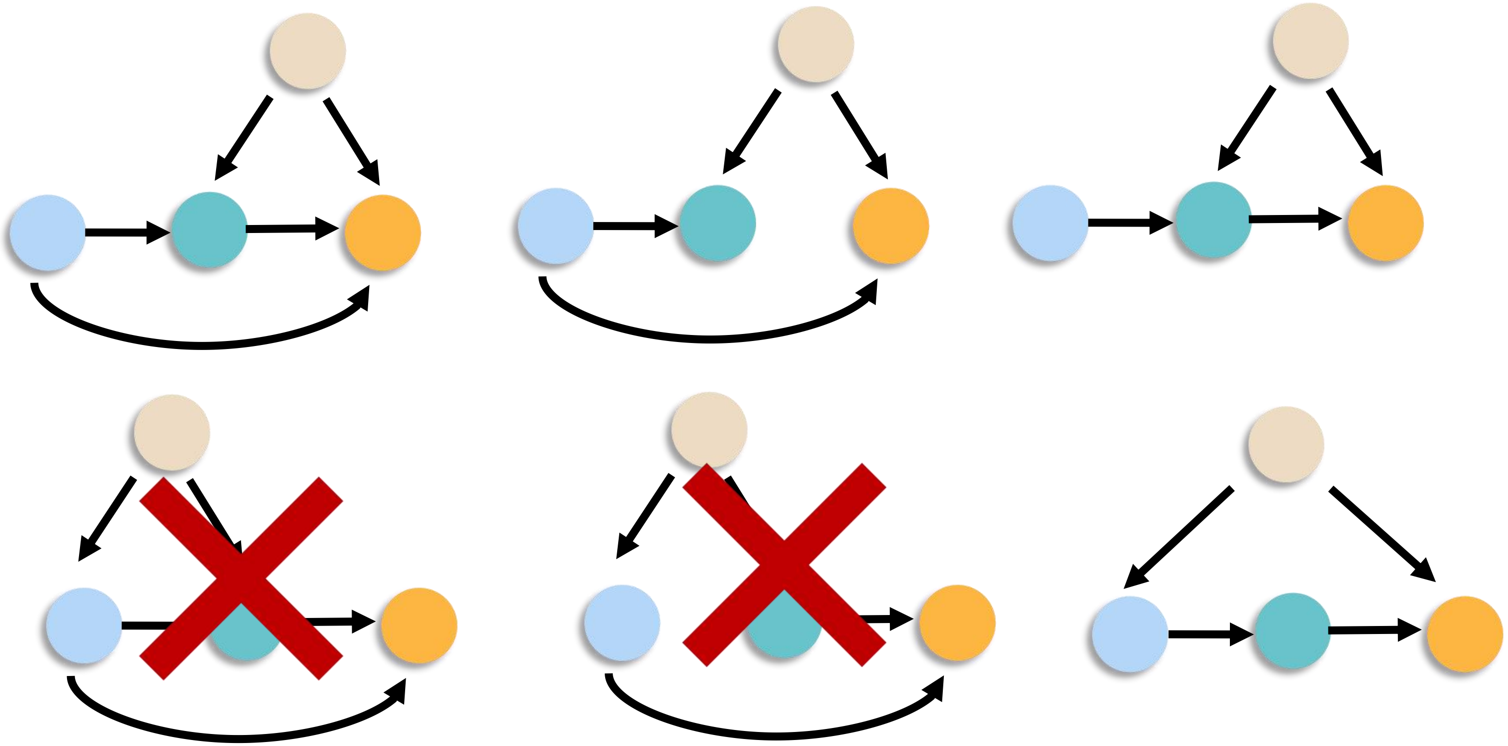
Matching Diversity and Complexity

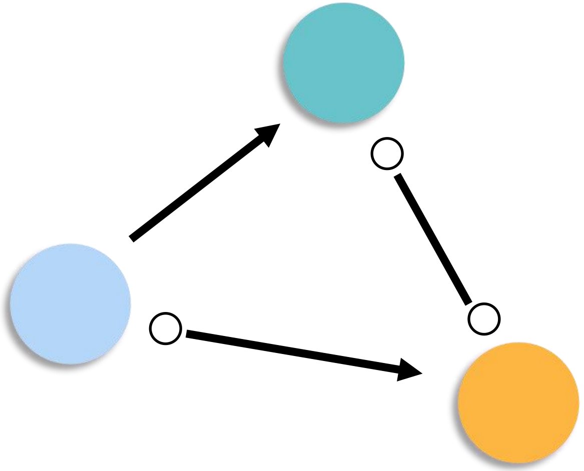




Markovian Equivalents

Matching Diversity and Complexity





Induce Path
Equivalents

Developments of Causal Discovery Algorithms

Markovian Equivalents

Induce Path Equivalents

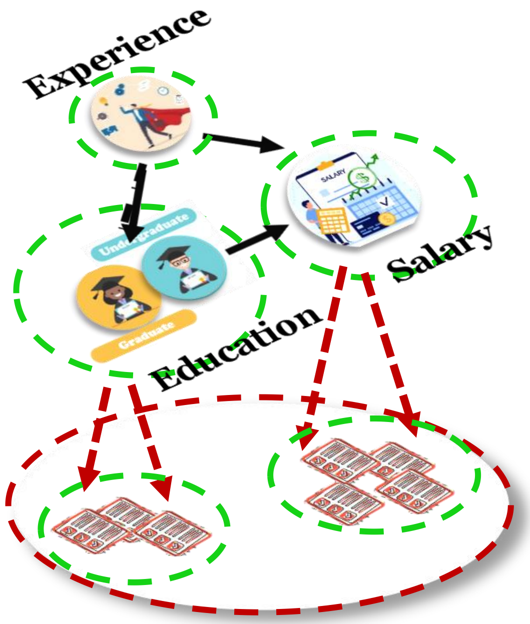
Spites-Glymour-Scheines
The SGS algorithm

Peter-Clark
The PC algorithm

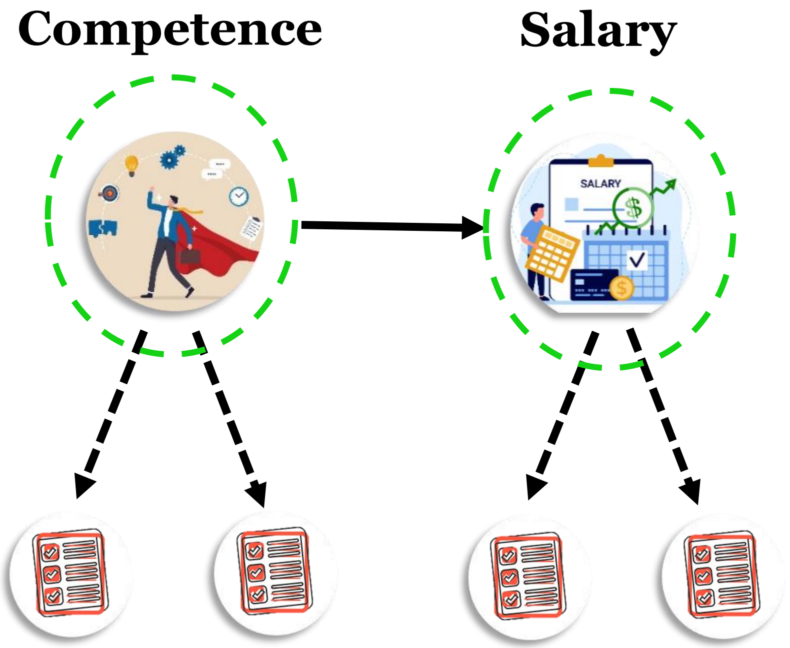
Verma, Pearl
The causal inference (CI) algorithm

Glymour Clark
The fast causal inference (FCI) algorithm

Back to the Education-Salary Example



Find out clusters



Intuition of Tetrad Constraints